RUDIMENTARY TREATISE

MENSURATION

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ERRATA.

Page 18, line 17, *for* "uinculum" *read* vinculum.
—— 29, in examples 4, 5, and 6, *for* "Prob. IV." *read* Prob. VI.
—— 30, in second formula, *for* "$C - c^2$" *read* $\overline{C - c}^2$
—— 32, last line but one, *for* "Prob. VIII." *read* Prob. IX.
—— 41, last line but three, *for* "train" *read* triangle.
—— 71, last line but fourteen, *for* "axes" *read* axis.

INTRODUCTION.

It will at once be seen that condensation of the materials produced by previous authors, and the introduction of a judicious selection of matter, adapted to the expanded intellect of the present age, are the proper requisites for a work on Mensuration. To this plan, the author trusts, from his long experience in engineering pursuits, that he has strictly adhered. In the first part on Practical Geometry, numerous examples are introduced, wherein the dimensions of certain parts are given to find the dimension of their corresponding parts, which has been rarely or never done by previous authors. This part is succeeded by a second part on the Mensuration of Lines; which is not added for the sake of novelty only, but because it seemed to be the natural order of a work of this kind. The third and fourth Parts treat of the Mensuration of Superficies and of Solids; while in all the three last named Parts the rules are not only given in words at length, in the usual way, but the same rules are expressed by Formulæ, together with other formulæ depending thereon, by which the rules receive considerable extension. Some of the rules and examples are taken *verbatim* from *Dr. Hutton's Mensuration;* for the author conceives that it would be disreputable to attempt, by *verbal alterations* in such rules, to give an air of originality to his work, as all other authors have done since Dr. H.'s time: the originality of this work consists in the new matter, every where added, to adapt it to the wants of modern times. Timber measuring and Artificers' work, the latter with considerable modern improvements are next introduced, with concise and practical methods of finding the surfaces and solidities of vaulted roofs, arches, domes, &c.

Concise, and the author trusts, clear systems of land sur-

veying, levelling, laying out railway curves and finding the contents of railway cutting, complete the work, and serve as an introduction to the author's *Land and Engineering Surveying*, which, though called a *Rudimentary Treatise*, in conformity with Mr. Weale's plan, contains everything, adapted to modern practice, that can be desired, an extension to this subject having been first given by the author, not found in any work previous to those written by him, see pages 174 and 198 of that work.

The demonstrations of all the rules and formulæ, in the four leading parts of the work will be found in *Dr. Hutton's Large Mensuration* and in the *Rudimentary Geometry;* the remainder of the demonstrations will be found in the author's *Railway Engineering*, or in his *Land and Engineering Surveying*.

Conic Sections and their solids are very briefly treated of in the four preceding parts of the work, and chiefly in as far as they may be useful to those who may intend to become excise officers, whose actual practice is best learnt from an experienced officer. Thus an extended article, such as is usually given by other authors is avoided, as not being generally useful to practical men. Those, who require extensive information on this subject, are referred to *Hann's Treatise on Conic Sections*, a most able and concise work, also in one of Mr. Weale's Rudimentary volumes.

The weights and dimensions of balls and shells may be found by Prob. VIII., Part IV., in conjunction with the Table and Rules for finding the specific gravities of bodies, if required.

The method of piling balls and shells, finding their number in a given pile, and the quantity of powder contained in a given shell or box, form no essential part of a work on mensuration, being only useful in an arsenal, and are, therefore, also omitted. The author has thus secured ample space for the discussion of subjects really useful to the great majority of students and practical men, in the compass of a volume less than half the size and one fifth of the price of the works of his predecessors; besides adding matter, adapted to the wants of modern times, not found in any existing work on mensuration.

The plan being thus briefly detailed, it will now be proper, previous to studying the following work, to give the

DIRECTIONS FOR BEGINNERS.

The beginner, for a first course, may omit the Problems beyond the thirty second in Practical Geometry, and Problems III., VIII., IX., XI., and XII., in the Mensuration of Lines, with the formulæ and examples depending on them. He may also omit all the formulæ, in the Mensuration of Superfices and Solids, with the examples depending on them, as well as the Problems beyond the tenth in the Mensuration of Solids, except it is required he should learn the method of gauging casks, in which case omit only the two last problems. But if he require an extensive knowledge of some or all the subjects, here treated of, he will do well to learn the use of such of the formulæ and the other parts omitted according to what he may require as a practical man.

T. B.

various English Railway Lines, showing a plain and easy method of taking out quantities of every description of Railway work and estimating them, and setting out work for the making of Railways generally. By E. V. GARDNER, C.E. In 1 vol., imperial 8vo., with 15 folding plates of works executed, containing tables and explanatory text, price 16s.

PLATES.

1. Section, part of the Keymer branch of the London and Brighton Railway.
2. Small barrel culverts on the Salisbury branch extension Railway.
3. Large culverts with wing-walls on the Salisbury branch extension Railway.
4. Open culverts on the Salisbury branch extension Railway
5. Bridge for stream at 9 miles 38 50 chains on the Portsmouth extension Railway.
6. Occupation road under the Farnham and Alton Railway.
7. Occupation bridge over the Farnham and Alton Railway.
8. Timber viaduct, Bricklayers' Arms branch of the London and Brighton Railway.
9. Inclined bridge for turnpike road over railway from Winchester to Southampton.
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12. Timber viaduct, with iron tension rods for any length.
13. Part of a Railway plan, and the section of it, with the curves set out, and offsets.
14. Sections of railway, in embankments and cuttings, with outside fencing, ditches, &c.
15. Part of the plan of the Syston and Peterborough Railway, with curves set off.

ENSAMPLES OF RAILWAY MAKING;

which, although not of English practice, are submitted, with practical illustrations, to Civil Engineers, and to the British and Irish public. In royal 8vo., 28 plates, extra cloth boards, price 12s.

CONTENTS:

Preliminary observations recommendatory of the adoption of a more economical mode of railway making in connection with the great lines already in operation, and a much further extension of a principle of railways for less advantageous traffic than the great lines afford, yet essential for the development of the resources of the country, by employment of material of a less cost. Landed proprietors would find their advantage in the improvement of their land by a quick transit, and, consequently, more ready sale for their produce. In some instances the material is on their estates.

Mechanical Works on the Utica and Syracuse Railroad, explanatory, with specification and cost of this, one of the best constructed railroads in the United States, made over swamps, creeks, and valleys, at a cost of £3600 per mile.

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1. Pont du Val-Benoit, Liege.
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7. Ditto, span of 60 feet.
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22, 23. Crossing plates for railroad.
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26. Viaduct under the Erie Canal, at Lodi.

RAILWAYS OF BELGIUM.

27. Sections of the Belgian Railway.
28. Section of the Vesdre Railway.

MENSURATION.

Mensuration treats of the various methods of measuring and estimating the dimensions and magnitudes of figures and bodies. It is divided into four parts, viz., Practical Geometry, and Mensuration of Lines, of Superficies, and of Solids, with their several applications to practical purposes.

PART I.

PRACTICAL GEOMETRY.

DEFINITIONS.

1. *A point* has no dimensions, neither length, breadth, nor thickness.

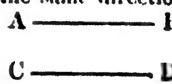
2. *A line* has length only, as A.

A

3. *A surface or plane* has length and breadth, as B.

4. *A right or straight line* lies wholly in the same direction, as A B.

5. *Parallel lines* are always at the same distance, and never meet when prolonged, as A B and C D.

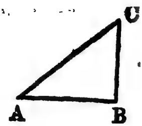


6. *An angle* is formed by the meeting of two lines, as A C, C B. It is called the angle A C B, the letter at the angular point C being read in the middle.

7. *A right angle* is formed by one right line standing erect or perpendicular to another; thus, A B C is a right angle, as is also A B E.

8. *An acute angle* is less than a right angle, as D B C.

9. *An obtuse angle* is greater than a right angle, as D B E.



11. ***A right angled triangle*** has one right angle, as A B C. The side A C, opposite the right angle, is called the hypothenuse; the sides A B and B C are respectively called the base and perpendicular.

12. ***An obtuse angled triangle*** has one obtuse angle, as the angle at B.

13. ***An acute angled triangle*** has all its three angles acute, as D.

14. *An equilateral triangle* has three equal sides, and three equal angles, as E.

15. *An isosceles triangle* has two equal sides, and the third side greater or less than each of the equal sides, as F.

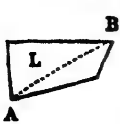
16. *A quadrilateral figure* is a space bounded by four right lines, and has four angles; when its opposite sides are equal, it is called a *parallelogram*.

17. *A square* has all its sides equal, and all its angles right angles, as G.

18. *A rectangle* is a right angled parallelogram, whose length exceeds its breadth, as B, (see figure to definition 2).

19. *A rhombus* is a parallelogram having all its sides and each pair of its opposite angles equal, as I.

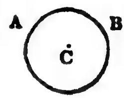
20. *A rhomboid* is a parallelogram having its opposite sides and angles equal, as K.



21. *A trapezium* is bounded by four straight lines, no two of which are parallel to each other, as L. A line connecting any two of its angles is called the *diagonal*, as A B.

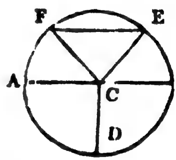
22. *A trapezoid* is a quadrilateral, having two of its opposite sides parallel, and the remaining two not, as M.

23. *Polygons* have more than four sides, and receive particular names, according to the number of their sides. Thus, a *pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; &c. They are called regular polygons, when all their sides and angles are equal, otherwise irregular polygons.



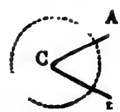
24. *A circle* is a plain figure, bounded by a curve line, called the circumference, which is everywhere equidistant from a point C within, called the centre.

25. *An arc of a circle* is a part of the circumference, as A B.



26. The *diameter of a circle* is a straight line A B, passing through the centre C, and dividing the circle into two equal parts, each of which is called a *semicircle*. Half the diameter A C or C B is called the *radius*. If a radius C B be drawn at right angles to A B, it will divide the semicircle into two equal parts, each of which is called a *quadrant*, or one fourth of a circle. A *chord* is a right line joining the extremities of an arc, as F E. It divides the circle into two unequal parts called *segments*. If the radii C F, C E be drawn, the space, bounded by these radii and the arc F E, will be the *sector of a circle*.

27. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees, and each degree into 60 minutes, each minute into 60 seconds, &c. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.



28. *The measure of an angle* is an arc of any circle, contained between the two lines which form the angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc:—thus the arc A B, the centre of which is C, is the measure of the angle A C B. If the angle A C B contain 42 degrees, 29 minutes, and 48 seconds, it is thus written 42° 29′ 48″.

PROBLEMS IN PRACTICAL GEOMETRY.

(In solving the five following problems only a pair of common compasses and a straight edge are required; the problems beyond the fifth require a scale of equal parts; and several of those beyond the thirteenth a line of chords: all of which will be found in a common case of instruments.)

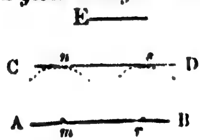
Problem I.

To divide a given straight line A B *into two equal parts.*

From the centres A and B, with any radius, or opening of the compasses, greater than half A B, describe two arcs, cutting each other in C and D; draw C D, and it will cut A B in the middle point E.

Problem II.

At a given distance E, *to draw a straight line* C D, *parallel to a given straight line* A B.



From any two points *m* and *r*, in the line A B, with a distance equal to E, describe the arcs *n* and *s*:—draw C D to touch these arcs, without cutting them, and it will be the parallel required.

Note. This problem, as well as the following one, is usually performed by an instrument called *the parallel ruler.*

Problem III.

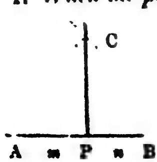
Through a given point r, to draw a straight line C D *parallel to a given straight line* A B.

From any point *n* in the line A B, with the distance *n r*, describe the arc *r m*:—from centre *r*, with the same radius, describe the arc *n s*:—take the arc *m r* in the compasses, and apply it from *n* to *s*:—through *r* and *s* draw C D, which is the parallel required.

Problem IV.

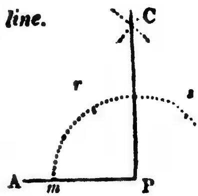
From a given point P *in a straight line* A B *to erect a perpendicular.*

1. *When the point is in or near the middle of the line.*



On each side of the point P take any two equal distances, P *m*, P *n*: from the points *m* and *n*, as centres, with any radius greater than P *m*, describe two arcs cutting each other in C; through C, draw C P, and it will be the perpendicular required.

2. *When the point* P *is at the end of the line.*



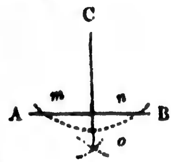
With the centre P, and any radius, describe the arc *m r s*;—from the point *m*, with the same radius, turn the compasses twice on the arc, as at *r* and *s*:—again, with centres *r* and *s*, describe arcs intersecting in C:—draw C P, and it will be the perpendicular required.

NOTE. This problem and the following one are usually done with an instrument called *the square*.

PROBLEM V.

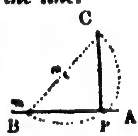
From a given point C *to let fall a perpendicular to a given line.*

1. *When the point is nearly opposite the middle of the line.*



From C, as a centre, describe an arc to cut A B in *m* and *n*;—with centres *m* and *n*, and the same or any other radius, describe arcs intersecting in *o*: through C and *o* draw C *o*, the perpendicular required.

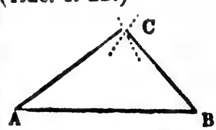
2. *When the point is nearly opposite the end of the line.*



From C draw any line C *m* to meet B A, in any point *m*;—bisect C *m* in *n*, and with the centre *n*, and radius C *n*, or *m n*, describe an arc cutting B A in P. Draw C P for the perpendicular required.

PROBLEM VI.

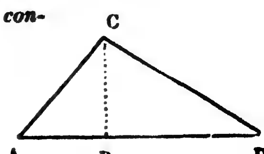
To construct a triangle with three given right lines, any two of which must be greater than the third. (Euc. I. 22.)



Let the three given lines be 5, 4 and 3 yards. From any scale of equal parts lay off the base A B = 5 yards; with the centre A and radius A C = 4 yards, describe an arc; with centre B and radius C B = 3 yards; describe another arc cutting the former arc in C:—draw A C and C B; then A B C is the triangle required.

PROBLEM VII.

Given the base and perpendicular, with the place of the latter on the base, to construct the triangle.

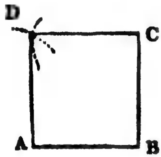


Let the base A B = 7, the perpendicular C D = 3, and the distance A D = 2 feet. Make A B = 7 and A D = 2;—at D erect the

perpendicular D C, which make = 3:—draw A C and C B; then A B C is the triangle required.

PROBLEM VIII.

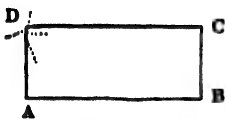
To describe a square, whose side shall be of a given length.



Let the given line A B be three feet. At the end B of the given line erect the perpendicular B C, (by Prob. IV. 2.) which make = A B:—with A and C as centres, and radius A B, describe arcs cutting each other in D: draw A D, D C, and the square will be completed.

PROBLEM IX.

To describe a rectangled parallelogram having a given length and breadth.



Let the length A B = 6 feet, and the breadth B C = 2. At B erect the perpendicular B C, and make it = 2:—with the centre A and radius B C describe an arc; and with centre C and radius A B, describe another arc, cutting the former in D: join A D, D C to complete the rectangle.

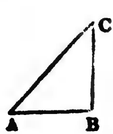
PROBLEM X.

The base and two perpendiculars being given to construct a trapezoid.

Let the base A B = 6, and the perpendiculars AD and B C, 2 and 3 feet respectively. Draw the perpendiculars A D, D C, as given above, and join D C, thus completing the trapezoid.

PROBLEM XI.

To construct a right angled triangle having a given base and perpendicular, and to find the hypothenuse.



Let the base A B = 6 feet, and the perpendicular B C = 8. Draw B C perpendicular to A B, and join A C; then A B C will be the triangle required, and A C being measured will be found = 10 feet.

Problem XII.

Having given the base and hypothenuse to construct the right angled triangle, and find the perpendicular.

(See figure to last Problem.)

Let A B = 6 feet, and A C = 10.—Draw the perpendicular B C indefinitely; take A C = 10 feet in the compasses, and with one foot on A apply the other to C; join A C, which completes the triangle, and B C will be found = 8 feet.

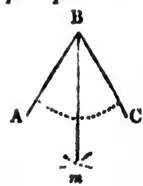
EXAMPLE.

A ladder 50 feet in length is placed with its foot 14 feet from a wall, the top of the ladder just reaching to the top of the wall; required the height of the wall.

Here 14 feet is the base of the right angled triangle, and 50 feet, = length of the ladder, is the hypothenuse, with which the triangle being constructed, the perpendicular will be found = 48 feet.

Problem XIII.

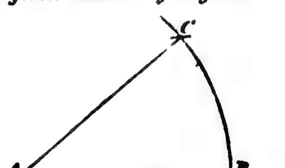
To divide a given angle A B C *into two equal parts.*



From the centre B, with any distance, describe the arc A C. From A and C, with one and the same radius, describes arcs intersecting in *m*. Draw the line B *m*, and it will bisect the angle as required.

Problem XIV.

To set off an angle to contain a given number of degrees.



Let the angle be required to contain 41 degrees. Open the compasses to the extent of 60° upon the line of chords, and, setting one foot upon A, with this extent, describe an arc cutting A B in B; then taking the extent of 41° from the same line of chords, set it off from B to C; join A C; then B A C is the angle required.

Problem XV.

To measure an angle contained by two straight lines.

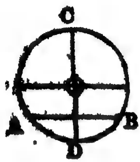
(See last figure.)

Let A B, A C contain the angle to be measured. Open the compasses to the extent of 60°, as before, on the line of chords, and with this radius describe the arc B C, cutting A B, A C

produced, if necessary, in B and C; then extend the compasses from B to C, and this extent, applied to the line of chords, will reach to 41°, the required measure of the angle B A C.

A right angle, or perpendicular, may be laid off by extending the arc B C, and setting off the extent of 90° thereon. Also an angle greater than 90° may be laid off, by still further extending the arc, and laying the excess of the arc above 90°, from the end of the 90th degree.

NOTE. Angles are more correctly and expeditiously laid off and measured by an instrument called the protractor, to be hereafter described.



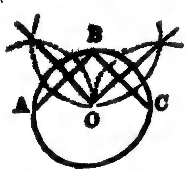
PROBLEM XVI.

To find the centre of a circle.

Draw any chord A B, and by Prob. I. bisect it perpendicularly with C D, which will be a diameter. Bisect C D in the point O, and that will be the centre.

PROBLEM XVII.

To describe the circumference of a circle through three given points A B C.



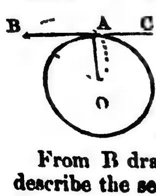
From the middle point B draw chords to the two other points A, C, bisect these chords perpendicularly by lines meeting in O, which will be the centre; then from the centre O, at the distance O A, or O B, or O C, describe the circle.

NOTE. In the same manner may the centre of an arc of a circle be found.

PROBLEM XVIII.

Through a given point A *to draw a tangent to a given circle.*

CASE I. *When* A *is in the circumference of the circle.*



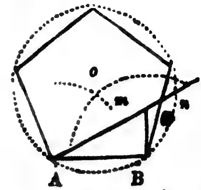
From the given point A, draw A O to the centre of the circle; then through A draw B C perpendicular to A O, and it will be the tangent as required.

CASE II. *When the given point is* B *not in the circumference.*

From B draw B O to the centre of the circle; and on B O describe the semicircle B A O, cutting the circle in A: then through B and A draw B A C, and it will be the tangent required.

PROBLEM XIX.

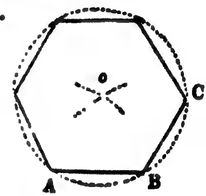
To make a regular pentagon on a given line A B.



Make B *m* perpendicular and equal to half A B; draw A *m*, and produce it till *mn* be equal to B*m*; with centres A and B, and distance B *n* describe arcs intersecting in *o*, which will be the centre of the circumscribing circle: then with the centre *o*, and the same radius, describe the circle; and about the circumference of it apply A B the proper number of times.

PROBLEM XX.

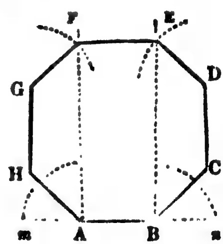
To make a hexagon on a given line A B.



With the distance A B, and the centres A and B, describe arcs intersecting in *o*; with the same radius and centre *o* describe a circle, which will circumscribe the hexagon; then apply the line A B six times round the circumference, marking out the angular points, and connect them with right lines.

PROBLEM XXI.

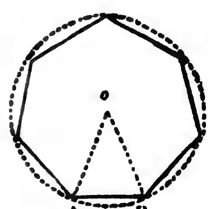
To make an octagon on a given line A B.



Erect A F and B E perpendicular to A B; produce A B both ways, and bisect the angles *m* A F and *n* B E with the lines A H and B C, each equal to A B; draw C D and H G parallel to A F or B E, and each equal to A B; with the distance A B, and centres G and D, cross A F and B E in F and E: then join G F, F E, and it is done.

PROBLEM XXII.

To make any regular polygon on a given line A B.



Draw A *o* and B *o* making the angles A and B each equal to half the angle of the polygon, by Prob. XIV., with the centre o and distance *o* A describe a circle: then apply the line A B continually round the circumference the proper number of times, and it is done.

NOTE. The angle of any polygon, of which the angles *o* A B and *o* B A are each one half, is found thus: divide the whole 360 degrees by the number of sides, and the quotient will be the angle at the centre *o*; then subtract that from 180 degrees, and the remainder will be the angle of the polygon, and is double of *o* A B or of *o* B A. And thus you will find the numbers of the following table, containing the degrees in the angle *o*, at the centre, and the angle of the polygon, for all the regular figures from 3 to 12 sides.

No. of sides.	Name of the Polygon.	Angle *o* at the centre.	Angle of the polygon.	Angle *o* A B or *o* B A.
3	Trigon	120°	60°	30°
4	Tetragon	90	90	45
5	Pentagon	72	108	54
6	Hexagon	60	120	60
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	$64\frac{2}{7}$
8	Octagon	45	135	67
9	Nonagon	40	140	70
10	Decagon	36	144	72
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	$73\frac{7}{11}$
12	Dodecagon	30	150	75

PROBLEM XXIII.

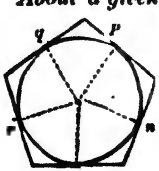
In a given circle to inscribe any regular polygon; or to divide the circumference into any number of equal parts.

(See the last figure.)

At the centre *o* make an angle equal to the angle at the centre of the polygon, as contained in the third column of the above table of polygons: then the distance A B will be one side of the polygon, which being carried round the circumference the proper number of times, will complete the figure. Or, the arc A B will be one of the equal parts of the circumference.

PROBLEM XXIV.

About a given circle to circumscribe any regular polygon.



Find the points *m*, *n*, *p*, &c., as in the last problem: to which draw radii *m o*, *n o*, &c., to the centre of the circle; then through these points *m*, *n*, &c., and perpendicular to these radii, draw the sides of the polygon.

EXAMPLE.

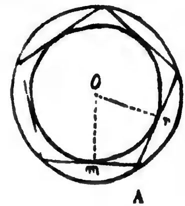
Let the radius of the given circle be 5 feet; then, having described a regular pentagon round it, the side of the figure

will be found = 7 feet $3\frac{1}{4}$ inches. If the figure to be described round the same circle be a regular hexagon, its side will be found = 5 feet $9\frac{1}{4}$ inches: and so on for any other regular polygons.

Problem XXV.

To find the centre of a given polygon, or the centre of its inscribed or circumscribed circle.

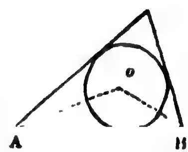
Bisect any two sides with the perpendiculars *m o*, *n o*, and their intersection will be the centre: then with the centre *o*, and the distance *o m*, describe the inscribed circle; or with the distance to one of the angles, as A, describe the circumscribing circle.



Problem XXVI.

In any given triangle to inscribe a circle.

Bisect any two of the angles with the lines A *o*, B *o*; and *o* will be the centre of the circle: then with the centre *o*, and radius the nearest distance to any one of the sides, describe the circle.



EXAMPLE.

Let the sides of the given triangle be 5, 4, and 3 feet; then, having inscribed a circle therein, its radius will be found = 1 foot.

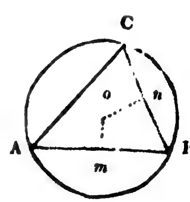
Problem XXVII.

About a given triangle to circumscribe a circle.

Bisect any two of the sides A B, B C, with the perpendiculars *m o*, *n o*; with the centre *o*, and distance to any one of the angles, describe the circle.

EXAMPLE.

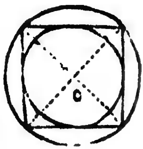
Let the sides of the given triangle be 15, 14, and 13 feet; then having described a circle about it, the radius will be found = 8 feet $1\frac{1}{4}$ inches.



Problem XXVIII.

In, or about, a given square, to describe a circle.

Draw the two diagonals of the square, and their inter-



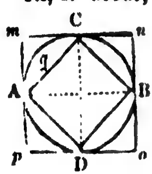
section *o* will be the centre of both the circles: then with that centre, and the nearest distance to one side, describe the inner circle, and with the distance to one angle, describe the outer circle.

EXAMPLE.

Let the side of the given square be 3 feet: then, having described circles in and about it, the radius of the former will be found = 1½ feet, and that of the latter = 2 feet 1½ inches nearly.

PROBLEM XXIX.

In, or about, a given circle, to describe a square, or an octagon.

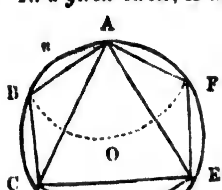


Draw two diameters A B, C D, perpendicular to each other; then connect their extremities, and that will give the inscribed square A C D B. Also through their extremities draw tangents parallel to them, and they will form the outer square *m n o p*.

NOTE. If any quadrant, as A C, be bisected in *q*, it will give one-eighth of the circumference, or the side of the octagon.

PROBLEM XXX.

In a given circle, to inscribe a trigon, a hexagon, or a dodecagon.



The radius of the circle is the side of the hexagon; therefore from any point A in the circumference, with the distance of the radius, describe the arc B O F: then is A B the side of the hexagon; and therefore carrying it six times round will form the hexagon, or will divide the circumference into six equal parts, each containing 60 degrees.—The second of these, C, will give A C the side of the trigon, or equilateral triangle A C E, and the arc A C one third of the circumference, or 120 degrees. —Also the half of A B, or A *n*, is one-twelfth of the circumference, or 30 degrees, which gives the side of the dodecagon.

NOTE. If tangents to the circle be drawn through all the angular points of any inscribed figure, they will form the sides of a like circumscribing figure.

EXAMPLE.

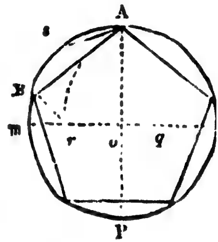
In a circle, the radius of which is 10 feet, inscribe a trigon, a hexagon, and a dodecagon.—Having measured a side of the se-

veral figures, that of the trigon will be found = 17 feet 4 inches, that of the hexagon 10 feet, and that of the dodecagon 5 feet 2 inches.

Problem XXXI.

In a given circle to inscribe a pentagon, or a decagon.

Draw the two diameters A P, *m n* perpendicular to each other, and bisect the radius *o n* at *q*: with the centre *q* and the distance *q* A, describe the arc A *r*; and with the centre A, and radius A *r*, describe the arc *r* B: then is A B one fifth of the circumference; and A B carried five times over will form the pentagon. Also the arc A B bisected in *s*, will give A *s* the tenth part of the circumference, or the side of the decagon.



Note. Tangents being drawn through the angular points, will form the circumscribing pentagon or decagon.

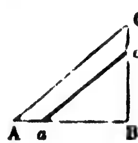
EXAMPLE.

In a circle, the radius of which is 10 feet, inscribe a pentagon and decagon.—Having measured a side of each of the figures, that of the pentagon will be found = 11 feet 9 inches, and that of the decagon = 6 feet 2 inches.

Problem XXXII.

To make a triangle similar to a given triangle A B C.

Make *a* B equal to the base of the required triangle; through *a* draw *a c* parallel to A C: then *a* B *c* is the triangle required.



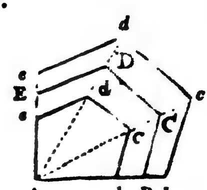
EXAMPLE.

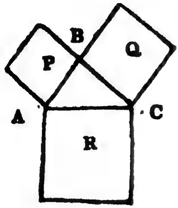
Let A B = 12, A C = 15 and B C = 9 feet; and the side *a* B of the required triangle = 8 feet.—Then having drawn *a c* parallel to A C, the side *a c* will be found = 10, and B *c* = 6 feet.

Problem XXXIII.

To make a figure similar to any other given figure A B C D E.

From any angle A draw diagonals to the other angles; take A *b* a side of the figure required; then draw *b c* parallel to B C, and *c d* to C D, and *d e* to D E, &c.





Problem XXXIV.

To make a square equal to two given squares P and Q.

Set two sides A B, B C, of the given squares, perpendicular to each other: join their extremities A C; so shall the square R, constructed on A C, be equal to the two P and Q taken together. (Euc. I., 47.)

Problem XXXV.

To make a square equal to the difference between two given squares, P, R.

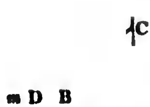
(See the last figure.)

On the side A C of the greater square, as a diameter, describe a semicircle; in which apply A B the side of the less square; join B C, and it will be the side of a square equal to the difference between the two P and R, as required.

Problem XXXVI.

To make a square equal to the sum of any number of squares taken together.

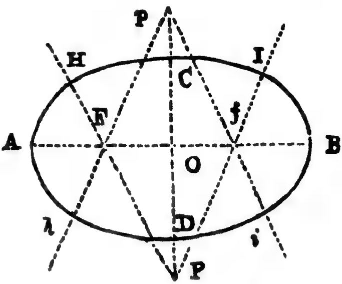
Draw two indefinite lines A *m*, A *n*, perpendicular to each other at the point A. On the one of these set off A B the side of one of the given squares, and on the other A C the side of another of them; join B C, and it will be the side of a square equal to the two together. Then take A D equal to B C, and A E equal to the side of the third given square. So shall D E be the side of a square equal to the sum of the three given squares.—And so on continually, always setting more sides of the given squares on the line A *n*, and the sides of the successive sums on the other line A *m*.



Problem XXXVII.

To construct a figure resembling an ellipse, by circular arcs from four centres.

On a line F *f*, of convenient length, describe two equilateral triangles F P *f*, F *p f*; prolong the sides of the triangles; join P *p*, as shown in the figure. With centres P, *p* and radius P H = *p h*, describe the arcs H I, *h i*, meeting the prolonged sides of the triangles, and such that the diameter C D

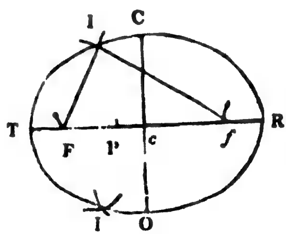


may be equal to the required width of the figure; with centres, F, f and radius H F = I f = &c., describe the arcs H A h, I B i, and the figure will be completed.

NOTE. If the longer diameter A B be not obtained of the required length by the above operation, the triangles F P f, F p f may be enlarged or diminished, or made isosceles, till by trials the proper dimensions are obtained.—This method of drawing the ellipse is practised by the picture-frame makers.

PROBLEM XXXVIII.

To describe a true ellipse.



Let T R be the transverse, C O the conjugate, and c the centre. With the radius T c and centre C, describe an arc cutting T R in the points F, f; which are called the two foci of the ellipse.

Assume any point P in the transverse; then with the radii P T, P R, and centres F, f, describe two arcs intersecting in I; which will be a point in the curve of the ellipse.

And thus, by assuming a number of points P in the transverse, there will be found as many points in the curve as you please. Then, with a steady hand, draw the curve through all these points.

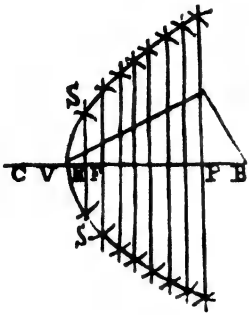
OTHERWISE,—WITH A THREAD.

Take a thread of the length of the transverse, T R, and fasten its ends with two pins in the foci F, f. Then stretch the thread, and it will reach to I in the curve: and by moving a pencil round, within the thread, keeping it always stretched, it will trace out the ellipse.

PROBLEM XXXIX.

To describe or construct a parabola.

V P being an absciss, and P Q its given ordinate; bisect P Q in A, join A V, and draw A B perpendicular to it;

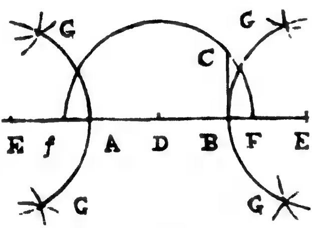


then transfer P B to V F and V C in the axis produced. So shall F be what is called the focus.

Draw several double ordinates S R S &c., perpendicular to V P. Then with the radii C R, &c. and the centre F, describe arcs cutting the corresponding ordinates in the points S, &c. Then draw the curve through all the points S, &c.

Problem XL.

To construct or describe an hyperbola.



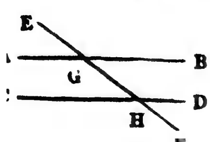
Let D be the centre of the hyperbola, or the middle of the transverse A B ; and B C perpendicular to A B, and equal to half the conjugate.

With centre D, and radius D C, describe an arc, meeting A B produced in F and *f*, which are the two focus points of the hyperbola.

Then assuming several points E in the transverse produced, with the radii A E, B E, and the centres *f*, F, describe arcs intersecting in the several points G ; through all which points draw the hyperbolic curve.

GEOMETRICAL THEOREMS.

(Necessary to be known by beginners).



Theorem I.

Angles vertically opposite are equal:—thus the angle A G E = angle H G B, and E G B = A G H. (Euc. I. 15.)

Theorem II.

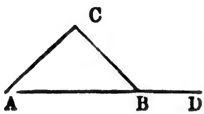
(See last figure.)

A right line E F, *cutting two parallel right lines* A B, C D,

Theorem III.

The greatest side of every triangle is opposite the greatest angle (Euc. I. 18.)

Theorem IV.



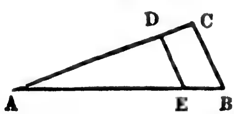
Let the side A B *of the triangle* A B C *be produced to* D, *the exterior angle* C B D *is equal to the interior angles at* A *and* C; *also the three interior angles of the triangle are equal to two right angles.* (Euc. I. 32.) Whence any two angles of a triangle being given the third becomes known.

Theorem V.

(See figure to Definition 11.)

Let A B C be a right angled triangle, having a right angle at B; then, *the square on the side* A C *is equal to the sum of the square on the sides* A B, B C. (Euc. I. 47.) Whence any two sides of a right angled triangle being given the third becomes known.

Theorem VI.



In any triangle A B C, let D E be drawn parallel to one of its sides, C B; then, A B *is to* A E *as* B C *is to* D E; and the triangles are said to be similar. (Euc. VI. 2.)

Theorem VII.

(See last figure.)

Let A B C, A E D be similar triangles; then, *the triangle* A B C *is to the triangle* A E D *as the square* A B *is to the square of* A E: that is, similar triangles are to one another in the duplicate ratio of their homologous sides. (Euc. VI. 19.)

Theorem VIII.

All similar figures are to one another as the squares of their homologous, or like, sides. (Euc. III. 20.)

Theorem IX.

All similar solids are to one another as the cubes of their like linear dimensions. (Euc. VI. 24.)

EXPLANATION OF THE PRINCIPAL MATHEMATICAL CHARACTERS USED IN THIS WORK.

The sign for equality $=$ is read "equal;" thus 12 inches $=$ 1 foot.

The sign for addition $+$ is read "*plus* or more;" thus $2 + 3 = 5$, $a + b$, &c.

The sign for subtraction $-$ is read "*minus* or less;" thus $5 - 2 = 3$, $a - b$, &c.

The sign for multiplication $\times$ is read "into;" thus $5 \times 3 = 15$, $a \times b$, or $a\,b$, &c.

The sign for division $\div$ is read "by;" thus $15 \div 3 = 5$, or $\frac{15}{3} = 5$, or $\frac{a}{b}$, &c.

The signs for proportion, as $:\ ::\ :$ "as, is to, so is, to;" thus as $2 : 5 :: 8 : 20$, or as $a : b :: c : d$, the fourth number being found by multiplying the second by the third, and dividing the first, as $\frac{5 \times 8}{2} = 20$, and $\frac{b\,c}{a} = d$.

The signs ( ) or $\{\ \}$ or $\overline{\quad}$ is called *uinculam* or brace; thus $(5 + 4) \times 2 = 9 \times 2 = 18$, or $\overline{5 + 4}\rceil\ 2 = 18$, $(a + b) \times c$, or $\overline{a + b}\rceil\ c$, &c.

The signs ^{2}, ^{3}, &c., placed above a quantity, represent respectively the square, cube, &c., of that quantity; thus $5^2 = 5 \times 5 = 25$, $5^3 = 125$, $\overline{3 + 4}\rceil^2 = 7^2 = 49$, $4\,(5 + 3)^2 = 4 \times 8^2 = 256$; and a^2, and a^3 represent the square and cube of a, also $(a + b)^2\,c^3$ signifies that the square of the sum of a and b is to be multiplied by the cube of c, &c.

The sign $\sqrt{\ }$ or $\sqrt[2]{\ }$ placed before a quantity, or $\frac{1}{2}$ placed above it, represents the square root of that quantity; thus $\sqrt{36} = 6$, $\sqrt{9 \times 16} = 12$, and $\sqrt{a \times b}$ or $\sqrt{a\,b}$ signifies the square root of the product of a and b, &c.

The sign $\sqrt[3]{\ }$ placed before a quantity, or $\frac{1}{3}$ placed above it, denotes the cube root of that quantity; thus

$\sqrt[3]{12 \times 2 \times 3 - 8}$, or $\sqrt[3]{(12 \times 2 \times 3 - 3)} = \sqrt[3]{72 - 8} = \sqrt[3]{64} = 4$, $\sqrt[3]{e\,\{(a + b)^2 - e\,d\}}$ denotes the cube root of the

difference of the square of the sum of a and b and the product of c and d multiplied into e. Also, the value of $\sqrt[3]{e\{(a+b)^2 - c\,d\}}$, when $a = 2$, $b = 7$, $c = 5$, $d = 9$, and $e = 6$ is $\sqrt[3]{6\{(2+7)^2 - 5 \times 9\}} = \sqrt[3]{6(81-45)} = \sqrt[3]{6 \times 36} = \sqrt[3]{216} = 6$.

PART II.

MENSURATION OF LINES.

THE MENSURATION OF LINES is applied to find the lengths of straight or curved lines, from the given lengths of other lines, on which these straight or curved lines depend.

TABLE OF LINEAL MEASURE.

Inches.	Feet.	Yards.	Poles.	Furlongs.	Mile.
12	1				
36	3	1			
198	$16\frac{1}{2}$	$5\frac{1}{2}$	1		
7920	660	220	40	1	
63360	5280	1760	320	8	1

$7\frac{23}{25} = 7{\cdot}92$ inches $= 1$ link.

22 yards $= 4$ poles $= 1$ chain of 100 links.

$69\frac{1}{2}$ English miles $= 60$ geographical miles $= 1$ degree.

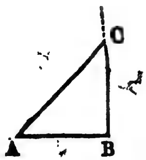
PROBLEM I.

To find one side of a right angled triangle, having the other two sides given.

The square of the hypothenuse is equal to both the squares of the two legs. (Euc. I. 47.) Therefore,

RULE I.—To find the hypothenuse; add the squares of the two legs together, and extract the square root of the sum.

RULE II.—To find one leg; subtract the square of the other leg from the square of the hypothenuse, and extract the square root of the difference.



Let A B C be a triangle, right angled at B; then by Theorem V. page 17, we shall have the following

FORMULÆ.

Put the base A B $= b$, the perpendicular B C $= p$, and the hypothenuse A C $= h$; then

$$h = \sqrt{b^2 + p^2},\ b = \sqrt{h^2 - p^2}, \text{ and } p = \sqrt{h^2 - p^2}.$$

EXAMPLE.

1. Required the hypothenuse of a right angled triangle, the base of which is 40 and the perpendicular 30 feet.

By Rule I.

```
  40     30
  40     30
 ----   ----
 1600   900
  900
 ----
 2500 (50 = hypothenuse A C.

   00
```

By first Formula.

$\sqrt{40^2 + 30^2} = 50 = $ A C.

NOTE. The student ought to solve this and all the following examples by geometrical construction, as in Problems XI. and XII.—Thus: make A B $= 40$ feet; draw B C $= 30$ feet perpendicular to A B, and join A C; then A C, being measured, will be found to be 50 feet.

The construction of example 2, will be as follows. Make A B $= 56$ feet, and perpendicular thereto draw B C indefinitely; take A C $= 65$ in the compasses, and with one foot on A apply the other foot to C; then B C, being measured will be found to be 33 feet.

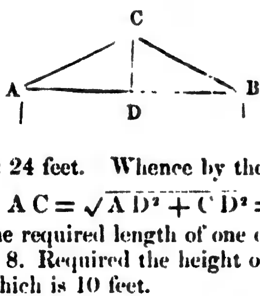
Remark. The triangle A B C, being for the purpose of illustrating the Problem generally, is not drawn to correspond to any of the dimensions given in the examples.

2. What is the perpendicular of a right angled triangle, whose base A B is 56, and the hypothenuse A C 65 feet?

```
   56      65
   56      65
  ----    ----
   336     325
  280     390
  ----    ----
  3136    4225
          3136
          ----
          1089 (33 feet = perpendicular B C.
            9
         -----
      63) 189
          189
```

6. A ladder of 50 feet long being placed in a street, reached a window 28 feet from the ground on one side; and by turning the ladder over, without removing the foot out of its place, it touched a moulding 36 feet high on the other side: required the breadth of the street. *Ans.* 76·1233335 *feet.*

7. The width of a house is 48 feet, and the height of ridge above the side walls 10 feet; required the length of one of the rafters.



In the annexed figure A B is the width of the house, or length of the tie-beam of the rafters A C, B C; and C D the height of the ridge or length of the king-post; and since D is the middle point of A B, we shall have A D = ½ A B = 24 feet. Whence by the first formula;

$$A\,C = \sqrt{A\,D^2 + C\,D^2} = \sqrt{24^2 + 10^2} = \sqrt{676} = 26 \text{ feet},$$

the required length of one of the rafters.

8. Required the height of an equilateral triangle, the side of which is 10 feet. *Ans.* 8 *ft.* 8 *in. nearly.*

9. The base of an isosceles triangle is 25 feet, and its two sides are each 32½ feet; required the perpendicular.

Ans. 30 *feet.*

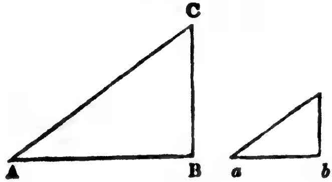
10. The diagonal of a square is 10 yards, required the length of one of its sides. *Ans.* 7 *yds.* 0 *ft.* 2¼ *in.*

11. A ladder, standing upright against a wall 100 feet high, was pulled out at the foot 10 feet from the wall; how far did the top of the ladder fall? *Ans.* 6 *inches nearly.*

12. The upright axle of the horse-wheel of a thrashing machine is placed with its centre 3½ yards from a wall; but the shafts of the axle are 5 yards in length, measured from the centre: how much of the wall must be removed to admit it to revolve? *Aas.* 7 *yds.* 0 *ft.* 5 *in.*

Problem II.

Having given any two of the dimensions of the figure A B C, *and one of the corresponding dimensions of a similar figure* a b c, *to find the other corresponding dimension of the last figure.*



Rule.—Let ABC, *a b c* be two similar triangles, then by Theorem VI., page 17.

$$AB : BC :: ab : ac, \text{ or } ab : ac :: AB : BC.$$

The same proportion holds with respect to the similar lineal parts of any other similar figures, whether plane or solid.

EXAMPLES.

1. The shadow of a cane 4 feet long, set perpendicularly, is 5 feet, at the same time that the shadow of a lofty tree was found to be 83 feet; required the height of the tree, both shadows being on level ground.

Let *b c* be the cane, and B C the tree, their shadows being respectively represented by *a b* and A B: the upper extremities of the cane and tree being joined with the extremities of their shadows, giving the parallel lines *a c*, A C for the directions of the sun's rays, and thus constituting similar triangles *a b c*, A B C: whence $ab : bc :: AB : BC$,

that is $5 : 4 :: 83 : 66\frac{2}{5}$

$$\begin{array}{r} 4 \\ \hline 5)332 \\ \hline \end{array}$$

$66\frac{2}{5}$ feet $=$ B C, the height of the tree.

2. The side of a square is 5 feet, and its diagonal 7·071 feet, what will be the side of a square, the diagonal of which is 4 feet.
Ans. 2 *ft.* 10 *in. nearly.*

3. In the ground plan of a building 120 feet long and 50 broad, the length, as laid down, is 10 inches; what must be its breadth? *Ans.* $4\frac{1}{6}$ *inches.*

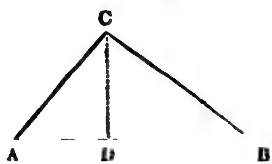
4. The scale of the Ordnance survey of Ireland is 6 chains to an inch, or 1 mile to $13\frac{1}{3}$ inches, what length of paper will be sufficient for the map of that country, its length being 300 miles.
Ans. $111\frac{1}{9}$ *yards.*

5. The lengh of the shadow of the Monument (London) is $151\frac{1}{2}$ feet, while the shadow of a post, 4 feet high, is 3 feet; required the height of the Monument. *Ans.* 202 *feet.*

Problem III.

The two sides and the base of a triangle (A B C) *are given to find the perpendicular* (C D).

Rule.—The segments of the base A D, D C must be first found. Let B C be the greater of the two sides, then B D will be the greater of the two segments. Then, as the base A B is to the sum of the sides B C + C A, so is the difference of the sides B C — C A to the difference of the segments of the base B D — D A. Half this difference, being added to and subtracted from half the base A B, will give respectively the segments B D, D A; though only one of the segments is required to be found. Now, either of the sides and its adjacent segments constitute a right angled triangle whence the perpendicular C D may be found by Rule II., Prob. I.



Formulæ.

Put A B $= a$, B C $= b$ and C A $= c$; then from the proportion in the Rule

$$BD - DA = \frac{b^2 - c^2}{a} \text{ ; whence}$$

$$BD = \tfrac{1}{2}\left(a + \frac{b^2 - c^2}{a}\right) \text{, and}$$

$$DA = \tfrac{1}{2}\left(a - \frac{b^2 - c^2}{a}\right).$$

Examples.

1. The three sides of a triangle are 42, 40 and 26 feet; required the perpendicular on the longest side.

By the Rule

$$AB : BC + CA :: BC - CA : BD - DA \text{, that is,}$$
$$42 : 66 :: 14 : 22 \text{, and}$$
$$\tfrac{1}{2}(42 - 22) = 10 \text{ feet} = AD$$

Or by the last Formula

$$DA = \tfrac{1}{2}\left(42 - \frac{40^2 - 26^2}{42}\right) = 10 \text{ feet,}$$

$$\text{and } CD = \sqrt{AC^2 - DA^2} = \sqrt{26^2 - 10^2} = 24 \text{ feet.}$$

2. The base of a triangle is 30, and the two sides 25 and 35; required the perpendicular. *Ans.* 24 *ft.* 6 *in. nearly.*

3. A house 21 feet in width, has a roof with unequal slopes, the lengths of which, from the eaves to the ridge, are 20 and 13 feet; required the height of the ridge above the eaves.

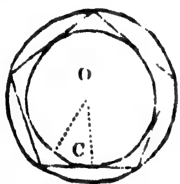
Ans. 12 *feet.*

NOTE. All the preceding examples may be readily solved by construction, by first laying down the triangles, as in Prob. VI., *Practical Geometry*, and then letting fall the perpendicular, as in Prob. V.

PROBLEM IV.

The side A B *of a regular polygon being given to find the radii* O C *and* O A *of its inscribed and circumscribed circles.*

RULE.—Multiply the side of the polygon by the number opposite its name in the following Table, in the column headed "Rad. Inscribed Circle," or in that headed "Rad. Circumsc. Circle," accordingly as the one or the other radius may be required.



FORMULÆ.

Let r and R be the radii of the inscribed and circumscribed circles respectively, q and p their respective tabular radii, and l = side of the polygon; then $r = l\,q$, and $R = l\,p$; also

$$l = \frac{r}{q} = \frac{p}{R}.$$

TABLE OF POLYGONS.

No. of Sides.	Name.	Rad. Inscribed Circle.	Rad. Circumsc. Circle.	Area.
3	Trigon or equi-triangle...	·2887	·5773	·4330
4	Tetragon or square........	·5000	·7071	1·0000
5	Pentagon....................	·6882	·8506	1·7205
6	Hexagon........	·8660	1·0000	2·5981
7	Heptagon....................	1·0383	1·1524	3 6339
8	Octagon......................	1·2071	1·3066	4·8284
9	Nonangon....................	1·3737	1·4619	6·1818
10	Decagon......................	1·5388	1·6180	7·6942
11	Undecagon..................	1·7028	1·7747	9·3656
12	Dodecagon..................	1·8660	1·9319	11·1962

EXAMPLES.

1. The side of a regular pentagon is 5 feet 1 inch, what are the radii of its circumscribed and inscribed circles?

Ans. 4 *ft.* 3·7 *in., and nearly* 3 *ft.* 6 *in.*

2. The side of an octagonal grass plot is $203\frac{1}{2}$ yards, and four walks are made therein, joining the middle of each of the opposite sides; required the united length of the four walks.

Ans. 1998 *yards.*

3. A circular grass-plot of 50 yards diameter is to be compassed by a regular octagonal iron paling, and the eight spaces, between the grass plot and the paling, to be planted with shrubs; required the whole length of the paling, and the greatest width of each of the eight spaces.

PROBLEM. V.

To find the diameter and circumference of a circle, the one from the other.

RULE I.—As 7 is to 22, so is the diameter to the circumference.

As 22 is to 7, so is the circumference to the diameter.

RULE II.—As 1 is to 3·1416, so is the diameter to the circumference.

As 3·1416 is to 1, so is the circumference to the diameter.

FORMULÆ.

Let $d =$ diameter, $c =$ cicumference, and $\pi = 3{\cdot}1416$;* then

$$c = d\,\pi, \text{ and } d = \frac{c}{\pi},$$

EXAMPLES.

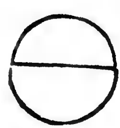
1. To find the circumference of a circle, whose diameter is 10.

By Rule I.

$7 : 22 :: 10 : 31\frac{3}{7}$

$$\begin{array}{r} 10 \\ \hline 7\,)\,220 \\ 31\frac{3}{7} \end{array}$$

or 31·42857 *Ans.*



* The true circumference of a circle, the diameter of which is unity, is 3·14159265358979, &c. This number has been determined by Machin to 100 places of decimals, by others to still more, and finally by Dr. Rutherford to 208 places; but these results are more curious than useful, since the first four or five decimals are quite sufficient for all practical purposes.

2

By Rule II. or first Formula $c = d\pi = 31{\cdot}416$, which is nearer the truth.

2. To find the diameter when the circumference is 50 feet.

By Rule I.

$$22 : 7 :: 50 : \frac{7 \times 25}{11} = \frac{175}{11} = 15\tfrac{10}{11} = 15{\cdot}9090 \textit{ Ans.}$$

By Rule II. or second Formula $d = \dfrac{c}{\pi} = 15{\cdot}9156$ feet.

3. If the diameter of the earth be 7958 miles, as it is very nearly, what is the circumference, supposing it to be exactly round? *Ans.* 25000·8528 *miles.*

4. To find the diameter of the globe of the earth, supposing its circumference to be 25000 miles. *Ans.* 7257 $\frac{3}{4}$ *nearly.*

5. Required the diameter of a coach wheel, that turns round 500 times in travelling a mile. *Ans.* 3 *ft.* 5·05 *in.*

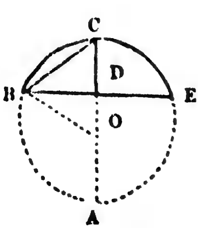
6. The driving wheel of a locomotive engine is 6 feet in diameter, how often does it turn in a second, when travelling at the rate of 60 miles in an hour? *Ans.* $4\frac{2}{3}$ *times nearly.*

PROBLEM VI.

The chord (B E) *and the height or versed sine* (C D) *of an arc* (B C E) *of a circle being given, to find the diameter* (A C) *and the chord of half the arc* (B C).

RULE.—Divide the square of half the chord B E, by the height C D; to the quotient add C D, and the sum will be the diameter A C; half of which is the radius B O or C O.

The chord B C of half the arc is found by Prob. I.



FORMULÆ.

Put C $= \frac{1}{2}$ chord of the whole arc $= \frac{1}{2}$ B E $=$ B D, $c =$ chord of $\frac{1}{2}$ the arc $=$ B C, $v =$ height or versed sine $=$ C D, and $d =$ diameter A C; then

$$d = \frac{C^2}{v} + v \ (1),\ c = \sqrt{C^2 + v^2} \ (2).$$

Also $d =$

EXAMPLE.

The chord of an arc is 48 feet, and its height 18; required the diameter of the circle of which the arc is a part.

By the Rule.

$$\begin{array}{r l}
 & 24 = \frac{1}{2}\,BE = BD \\
 & \underline{24} \\
CD = 18) & \underline{576} \\
 & 32 \\
 & \underline{18} = CD \\
 & 50 \text{ feet} = AC.
\end{array}$$

whence the radius B O = 25 feet.

2. The span (chord) of the iron arch of Sunderland bridge is 240 feet, and the rise (height) of the crown of the arch 34 feet: with what radius was the arch drawn?

By Formula (1)

The diameter $= \frac{120^2}{34} + 34 = 440{\cdot}41$ feet,

whence the required radius $= 440{\cdot}41 \div 2 = 220{\cdot}205 = 220$ feet $2\frac{1}{2}$ inches.

3. On a parliamentary map of 4 chains to an inch, the chord of a railway curve measured 40 inches, and its height 5 inches, required the radius of the curve both on the map and on the ground.

The diameter $= \frac{20^2}{5} + 5 = 85$ inches, whence

the radius $= 85 \div 2 = 42\frac{1}{2}$ inches on the map.

And, since the scale of the map is 4 chains to an inch, we shall have $42\frac{1}{2} \times 4 = 170$ chains $= 2\frac{1}{8}$ miles, the radius of the curve on the ground.

4. The chord of the whole arc is 48 feet, and its height 7, required the chord of half the arc.

By Formulæ (2).

$c = \sqrt{C^2 + v^2} = \sqrt{24^2 + 7^2} = 25$ feet, the required chord.

5. The chord of half the arc of a bridge is 24 feet and the rise of the crown of the arch 16 feet, required the radius of the circle of which the arch is a part.

Ans. By formula (3) the diameter is found = 36 feet, whence the required radius is 18 feet.

6. A circular grass plot of 100 yards diameter is cut by a walk through the centre, this walk is cut at right angles by another walk through the middle of the radius; required the length of the last named walk.

By transposing formula (1) $C = \sqrt{v\,(d - v)} = \sqrt{25\,(100 - 25)}$ $= 43{\cdot}3$ yards the double of which is the length of the walk.— The same result may be obtained from the right angled triangle B D O.

7. The rise of the circular arch of a bridge is 12 feet and the radius of the whole circle is 100 feet; required the distance from the spring of the arch to the crown, viz., the chord of half the arch. *Ans.* 49 *feet nearly.*

Problem VII.

To find the length of any arc of a circle.

Case I.—*When the degrees in the arc and the radius are given.*

Rule I.—As 180° is to the number of degrees in the arc, so is 3·1416 times the radius to its length.

Rule II.—From 8 times the chord of half the arc subtract the chord of the whole arc, and take $\frac{1}{3}$ of the remainder for the length of the arc nearly.

FORMULÆ. (*See last figure*).

Put $r =$ radius B O, $\Delta = 180°$, $\delta =$ degrees in the arc B E, and $\pi = 3·1416$, and $l =$ length of the arc; then

$$l = \frac{r\,\delta\,\pi}{\Delta}, \text{ and } r = \frac{l\,\Delta}{\delta\,\pi}.$$

EXAMPLES.

1. To find the length of an arc of 30 degrees, the radius being 9 feet.

By Rule I.

$$\begin{array}{r} 3·1416 \\ 9 \\ \hline \end{array}$$

180 : 30 : : 28·2744 : 4·7124 feet.

By first Formula $l = \dfrac{9 \times 30 \times 3·1416}{180} = \dfrac{3 \times 3·1416}{2} = 4·7124$

2. The length of the arc of a circle of 30 degrees is 9 feet 5 inches, required its radius.

Ans. By the second formula, 18 *feet nearly.*

3. The chord B E of the whole arc being 4·65374 feet, and the chord B C of the half arc 2·34947; required the length of the arc.

By Rule II.

$$\begin{array}{r} 2·34947 \\ 8 \\ \hline 18·79576 \\ 4·65874 \\ \hline 3)14·13702 \\ \hline \end{array}$$

Ans. 4·71234 *feet.*

3. Required the length of an arc of 12 degrees 10 minutes, or $12\frac{1}{6}$ degrees, the radius being 10 feet.

By Rule I., 2·1234 *feet, Ans.*

4. Required the length of the iron arch, in example 2, Prob. IV.

First, the chord of $\frac{1}{2}$ the arch, or distance from spring to crown, by Formula 2, Prob. IV., will be found 124·724 feet. Whence, by Rule II. of this Problem, we shall have the required length of the arch = 252 feet 7 inches.

5. Find the length of one of the arcs of the six equal segments of an iron girder, the whole span of the arch being 120 feet, and the radius 180. *Ans.* 20 *feet* 4·67 *inches.*

Rule II. is not sufficiently accurate for finding the length of the arc, when it is greater that $\frac{1}{4}$ of the circumference of the circle: in such cases, (see figure to Prob. IV.) the chord of $\frac{1}{4}$ of the arc B C E = chord of $\frac{1}{2}$ the arc B C (not shewn in the figure) must be found by the formula.

$$\text{Chord of } \tfrac{1}{4} \text{ of arc B C E} = \sqrt{\tfrac{1}{2}\, d\,(d - \sqrt{d^2 - c^2})}.$$

in which d and c are the same as in Prob. IV.; after which Rule II. may be applied with sufficient accuracy to find the length of the $\frac{1}{2}$ arc B C, which, being doubled, will give the whole length B C E.

6. Required the length of a circular iron girder, the span (B E) of which is 48 feet, and the rise (C D) at the crown 18 feet.

By Formulæ 1 and 2, Problem IV., d = A C is found = 50 feet, and c = B C = 30; whence, by the formula just given, the chord of $\frac{1}{4}$ of arc B C E = $\sqrt{25\,(50 - \sqrt{50^2 - 30^2})}$ = 15·8113, and by Rule II., (15·8113 × 8 − 30) ÷ 3 = 32·1635 feet = arc B C, the double of which is 64·3270 feet = the required length of the arch B C E. But by using Rule II., without the above formula, the length of the arch is found to be 64 feet, or nearly 4 inches short of its more accurate length, as previously found.

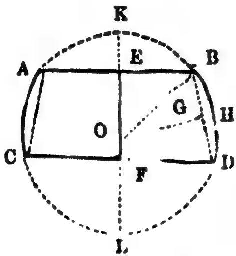
NOTE. The true method of finding the length of an arc of a circle is to find the natural sine of the angle B O D (figure to Prob. V.) and its corresponding number of degrees, minutes, &c., which, being doubled, give the angular measure of the whole arc B C E; whence the length of the arc may be accurately found by Rule I. But the first part of this operation is the province of Trigonometry; moreover, sufficient accuracy for all practical purposes may be ob-

tained by Rule II. for arcs less than a quadrant; and like accuracy may be secured by means of the formula used in Example 6, in cases where the arc approaches near to a semicircle.

Remark.—When the arc is greater than a semicircle, the remaining part of the circumference must be found by Rule II., with the help of the formula used in Example 6, if necessary. This remark does not apply to cases where the degrees of the arc are given, which are solved by Rule I.

Problem VIII.

To find the diameter of a circular zone, its two parallel chords A B, C D, *and its breadth* E F, *being given.*



This and the following Problems may be omitted by the student, as not being much required in practice.

FORMULÆ.

Let C and c be the half chords C F and A E respectively, b the breadth E F, and d the diameter K L = twice radius O B; then

$$d = \surd\left\{b^2 + 2(C^2 + c^2) + \left(\frac{C^2 - c^2}{b}\right)^2\right\}.$$

Also $AC = BD = \surd(b^2 + C - c^2)$, and

$$GH = \tfrac{1}{2}d - \tfrac{1}{2}\surd\left\{(C + c)^2 + \left(\frac{C^2 - c^2}{b}\right)^2\right\}$$

EXAMPLES.

1. The parallel sides of a circular zone are 6 and 8 feet, and its breadth 7 feet; required the diameter of the circle.

By the first Formula the diameter

$$d = \surd\left\{7^2 + 2(4^2 + 3^2) + \left(\frac{4^2 - 3^2}{7}\right)^2\right\} = \surd\overline{49 + 50 + 1} = 10 \text{ ft.}$$

2. Find the chord B D and the height G H of the zone in the preceding Example.

Here, the diameter d is first found, as above; then by the second and third Formulæ,

$$BD = \surd(7^2 + 4 - 3^2) = \surd\overline{49 + 1} = 7{\cdot}07 \text{ feet, and}$$

$$GH = \tfrac{1}{2}\,10 - \tfrac{1}{2}\surd\left\{(4 + 3)^2 + \left(\frac{4^2 - 3^2}{7}\right)^2\right\} = 5 - \tfrac{1}{2}\surd\overline{49 + 1}$$

3. The parallel chords of a zone are the same as in the Example 1., and its breadth 1 foot; required the diameter.

14 *feet.*

NOTE. In this example the two chords are both on the same side of the centre of the circle.

4. The two parallel chords of a circular zone are 16 and 12 feet, and the diameter of the circle 20 feet; required the breadth of the zone. *Ans.* 14 *feet.*

NOTE 1. The breadth of the zone, in this example, is found by squaring and transposing the first formula, whence there results a quadratic equation, from which the value of b is found.

NOTE 2. When the chord B D = A C, and the height G H have been found, the lengths of the equal arcs A C, B D are found by the Prob. VI.

PROBLEM IX.

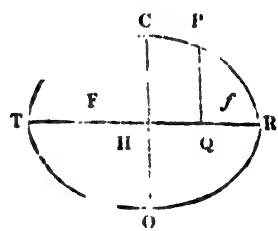
In an ellipse are given any three of the four following parts to find the fourth, viz. the transverse axis T R, *the conjugate axis* C O, *the abscissa* H Q, *and the ordinate* P Q.

FORMULÆ.

Put a = semitransverse = H R, b = semiconjugate = C H, x = abscissa = H Q, and y = ordinate = P Q; then

$$x = \frac{a}{b}\sqrt{b^2-y^2},\ y = \frac{b}{a}\sqrt{a^2-x^2},$$

$$a = \frac{b\,x}{\sqrt{b^2-y^2}},\ \text{and } b = \frac{a\,y}{\sqrt{a^2-x^2}}$$



Also the focial distance from the centre

$$\text{H F} = \text{H}f = \sqrt{a^2-b^2}.$$

EXAMPLES.

1. The transverse axis is 30, the conjugate 20, and the abscissa 3 feet.

By the second formula,

$$\text{P Q} = y = \tfrac{10}{15}\sqrt{15^2-3^2} = 9{\cdot}798 \text{ feet.}$$

2. The transverse T R = 70 feet, the conjugate C O = 50, and the ordinate P Q = 20; required the abscissa H Q.

Ans. By the first formula, H Q = 21 *feet.*

3. The transverse is 180 inches, the ordinate 16, and the abscissa 54; required the conjugate.

Ans. By the fourth formula, the conjugate = 40 *inches.*

4. If the conjugate be 50 feet, the ordinate 20, and the abscissa 21; what is the transverse?

Ans. By the third formula, the transverse = 70 *feet.*

5. The transverse T R = 100 yards, and the congugate C O = 60; required the distance of the foci F f from the centre H.

Ans. By the last formula, H F = H f = 40 *yards.*

6. The ratio of the major and minor axes of the earth's orbit is as 1 to n, the former being about 190,000,000 miles $= 2a$, How much is the earth nearer to the sun in winter than in summer?

Ans. The distance here required is twice the focial distance from the centre of the earth's elliptical orbit, which, by the last formula is found to be $2a\sqrt{1-n^2}$.

7. Required the distance of the foci of an elliptical section, passing through the poles of the earth, the earth's axes being 7926 and 7899 miles.

Ans. 654 *miles, or* 327 *miles each from the earth's centre.*

Problem X.

The axes of an ellipse are given to find its circumference.

Rule I.—Multiply the sum of the two axes by 3·1416, and the product will give an approximate length of the circumference, *which will be found near enough for most practical purposes.*

Rule II.—To half the sum of the two axes add the square root of half the sum of their squares, and multiply half the sum by 3·1416 for the circumference *very nearly.*

FORMULÆ *(see last figure).*

Let $2a$ and $2b$ represent the axes, as in the last problem, and $\pi = 3\cdot1416$; then,

$$\text{Circumf.} = \pi\,(a+b), \text{ or } = \tfrac{1}{2}\pi\left(a+b+\sqrt{2(a^2+b^2)}\right).$$

EXAMPLES.

1. The axes of an ellipse are 15 and 10 feet; required the circumference by Rule I.

Ans. 39 *feet* 3 *inches.*

2. The axes being the same as in the last example; required the circumference by Rule II.

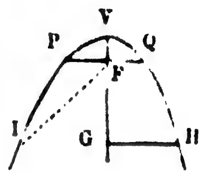
Ans. 39 *feet* 8 *inches nearly.*

3. Find the meridional circumference of the earth, the axes being as given in the last example of Prob. VIII.

Ans. 24,858 *miles nearly.*

Problem XI.

In a parabola* I V H, *the focus of which is* F, *any two of the three following parts, viz. the parameter* P Q, *the obscissa* V G, *and the ordinate* G H *being given, to find the third part.



FORMULÆ.

Put P Q = parameter = p, V G = abscissa = x, and C H = ordinate = y; then

$$x = \frac{y^2}{p},\ y = \sqrt{p\,x},\ \text{and}\ p = \frac{y^2}{x}.$$

EXAMPLES.

1. The parameter P Q of a parabola is 50, and its ordinate G H = 60 feet, required the abscissa V G.

Ans. By the first formula $x = G\,H = \frac{60^2}{50} = 72$ *feet.*

2. The parameter of a parabola is 10, and its ordinate 4: required the abscissa. *Ans.* 1·6.

3. The abscissa of a parabola is 4, and its corresponding ordinate 10; required the parameter. *Ans.* 25.

Problem XII.

To find the length of the arc of a parabola, its ordinate and abscissa being given. (*See last figure.*)

FORMULÆ.

Let x and y represent the same parts, as in the last Problem: then, The $\frac{1}{2}$ arc V H = $\sqrt{\frac{4}{3}x^2 + y^2}$ nearly.

EXAMPLES.

1. Required the half arc V Q of a parabola, V F being = 3 feet, and F Q = 6.

Ans. V Q = $\sqrt{\frac{4}{3}3^2 + 6^2}$ = 6 *feet* $11\frac{1}{8}$ *inches.*

2. The abscissa is 2, and the ordinate 6; required the length of the half arc of the parabola. *Ans.* 6·4291.

Note 1. The parabola is the path of projectiles *in vacuo*; it is also used in the astronomical theory of comets.

Note 2. The student who wishes for further information concerning this curve, as well as concerning the ellipse and hyperbola, may consult the various works on *conic sections.*

MENSURATION OF SUPERFICIES OR SURFACES.

The area of any surface is estimated by the number of squares in that surface, without regard to its thickness; the side of those squares being one inch, one foot, one yard, &c. Hence the area is said to be so many square inches, or square feet, or square yards, &c.

A TABLE OF SQUARE MEASURE.

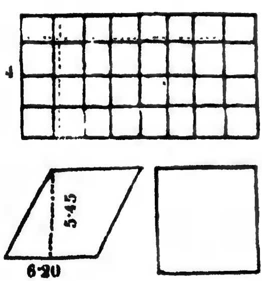
Sq. Inches.	Sq. Feet.	Sq. Yards.	Sq. Poles.	Sq. Rods.	Acres.	Sq. Mile.
144	1					
1,296	9	1				
39,204	272¼	30¼	1			
	1,089	1,210	40	1		
	43,560	4,840	160	4	1	
		3,097,600	102,400	2,560	640	1

PROBLEM I.

To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboid.

RULE.—Multiply the length by the breadth or perpendicular height, and the product will be the area.

FORMULÆ.



Let l = length of the figure, b = its breadth, and A = its area (*which also represents the areas in all the following Problems*); then

$$A = lb, \text{ also } l = \frac{A}{b}, \text{ and } b = \frac{A}{l}.$$

When the figure is a square, then the length is equal to the breadth, which put $= s =$ side of the square; then

$$A = s^2, \text{ and } s = \sqrt{A}.$$

EXAMPLES.

1. The length of a rectangular board is 7 feet, and its breadth 4 feet; required its area in square feet. (See first figure.)

By the Rule. $7 \times 4 = 28$ square feet, the area required.

2. The side of a square is 18 inches; required its area in square feet. (See last figure.)

$$
\begin{array}{r|l}
 & 18 \\
 & 18 \\
\hline
144\left\{\begin{array}{l}12\\12\end{array}\right. & \begin{array}{|l}324\\\hline 27\end{array} \\
\hline
\end{array}
$$

$2\frac{1}{4}$ square feet, the area required.

3. Find the area of a rhombus, the length of which is 6·2 feet, and its perpendicular breadth 5·45. (See second figure.)

Ans. 33·79 = $33\frac{3}{4}$ *square feet nearly.*

4. The length of a table is 7 feet 8 inches, and its breadth 3 feet 10 inches; required its area.

feet.	in.	
7	8	
3	10	
23	0	
6	4	8
29	4	8

Here the operation is performed by duodecimals, and the area is found to be 29 square feet, 4 inches or 12ths, and 8 parts or $\frac{8}{144}$ths.

5. What length must be cut off a rectangular board, the breadth of which is 9 inches, to make a square yard?

A square yard contains 1296 square inches, whence by the second formula.

144 inches = 12 feet, the length required.

6. How many square feet of deal will make a box 6 feet long, 5 broad, and 2 feet 8 inches deep?

Ans. 116 *square feet* 2′ 8″.

7. How many square yards are contained in a floor 23 feet long $14\frac{1}{2}$ feet wide? *Ans.* $37\frac{1}{18}$ *square yards.*

8. The base of the largest Egyptian pyramid is a square, the side of which is 693 feet; required the number of acres it occupies. *Ans.* 11*a.* 0*r.* 4*p.*

9. A square court yard is 42 feet long, and 23 feet $10\frac{1}{2}$ inches broad; what did it cost paving at 4*s.* 10*d.* per square yard.

Ans. £26 18*s.* $6\frac{1}{4}$*d.*

10. Required the side of a square, the area of which is 500 square feet.

By the fourth formula s = side of the square = $\sqrt{A}$, that is $s = \sqrt{500} = 22{\cdot}3607$ feet = 22 feet $4\frac{1}{4}$ inches nearly.

11. What is the side of a square the area of which is an acre *Ans.* 69·6 *yards nearly.*

12. A square in a city contains $6\frac{1}{4}$ acres of ground, required the side of the square. *Ans.* 17392 *yards.*

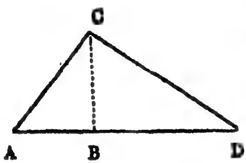
PROBLEM II.

To find the area of a triangle.

RULE I.—Multiply the base by the perpendicular height, and take half the product for the area.

II.—When the three sides only are given: add the sides altogether, and take half the sum; from the half sum subtract each side separately; multiply the half sum and the three remainders continually together; and take the square root of the last product for the area of the triangle.

FORMULÆ.



Let the base A B $= b$, and the perpendicular C D $= p$; then

$A \qquad \frac{2A}{b} \quad \frac{2A}{p}$.

When all the three sides of the triangle are given, let them be represented by a, b and c, and their half sum by s; then

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

EXAMPLES.

1. Let the base AB = 42 feet, and the perpendicular C D = 33 feet; required the area in square yards.

By Rule I. $42 \times 33 \div 2 = 693$, and $693 \div 9 = 77$ square yards.

2. To find the number of square yards in a triangle, the sides of which are 13, 14 and 15 feet.

By Rule II.

	13			21
	14			6
	15			126
	2)42			7
½ sum	21	21	21	882
	13	14	15	8 9
remainders	8	7	6	7056(84 *square feet*,
				64 — 9⅓ *sq. yds. Ans.*
				164)656
				656

3. The base of a triangle is 40, and its perpendicular 30 feet; required the area in square yards. *Ans.* 66⅔ *square yards.*

4. Find the area of a triangle, the three sides of which are 20, 30 and 40 feet. *Ans.* 32·27 *square yards.*

5. The base of a triangle is 49 and its height 25¼ feet, how many square yards does it contain?

Ans. 68·736 *square yards.*

6. The base of a triangle is 18 feet 4 inches, and its height 11 feet 10 inches; required the area. *Ans.* 108 *feet,* 5′. 8″.

7. The hypothenuse of a right angled triangle is 102½ feet, and its base 100; required the area in square yards.

Ans. 125 *square yards.*

8. The side of an equilateral triangle is 5·1 feet, required the area. *Ans.* 11·2626 *square feet.*

9. The base of a triangle is 121 yards; required its perpendicular, when it contains an acre of land. *Ans.* 80 *yards.*

10. The equal sides of an isosceles triangle are each 50 feet, and its base 28; how many square yards does it contain?

Ans. 74⅔ *square yards.*

PROBLEM III.

To find the area of a trapezoid.

Add together the two parallel sides; multiply that sum by the perpendicular distance between them, and take half the product for the area.

EXAMPLES.

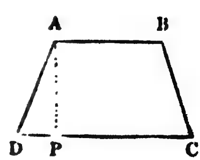
1. In a trapezoid the parallel lines are A B 7·5, and D C 12·25, also the perpendicular distance A P is 15·4 feet; required the area.

12·25
7·5

19·75
15·4

7900
9875
1975

2)304·150
152·075 *square feet, Ans.*



2. How many square feet contains the plank, whose length is 12 feet 6 inches, the breadth at the greater end 1 foot 8 inches, and at the less end 11 inches? *Ans.* 13$\frac{1}{24}$ *feet.*

3. Required the area of a trapezoid, the parallel sides being 21 feet 3 inches and 18 feet 6 inches, and the distance between them 8 feet 5 inches. *Ans.* 167 *square feet,* 3′ 4″ 6‴.

Problem IV.

To find the area of a trapezium.

Case I.—*For any trapezium.*

Divide it into two triangles by a diagonal; then find the areas of these triangles, and add them together.

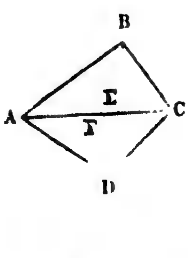
Or, if two perpendiculars be let fall on the diagonal, from the other two opposite angles, the sum of these perpendiculars being multiplied by the diagonal, half the product will be the area of the trapezium.

Case II.—*When two opposite angles are supplements of each other.*

Add all the four sides together, and take half the sum: next subtract each side separately from the half sum; then multiply the four remainders continually together, and take the square root of the last product for the area of the trapezium.

EXAMPLES.

1. To find the area of the trapezium A B C D, the diagonal A C being 42, the perpendicular B E 18, and the perpendicular D F 16.




```
   18

   34 Sum
   42
   --
   68
  136

2)1423
   714 Ans.
```

2. In the trapezium A B C D, the side A B is 15, D C 13, C D 14, A D 12, and the diagonal A C is 16: required the area.

A C	16				A C	16			
A B	15				C D	14			
B C	13				A D	12			
	2)44					2)42			
	22	22	22	half sum		21	21	21	half sum
	16	15	13			16	14	12	
	6	7	9			5	7	9	
	7					7			
	42					35			
	9					9			
	378					315			
	22					21			
	756					315			
	756					630			

$\sqrt{8316} = 91 \cdot 1921$ $\qquad$ $\sqrt{6615} = 81 \cdot 3326$

The triangle A B C...... 91·1921
The triangle A B C...... 81·3326

The trapezium A B C D 172·5247, *Ans.*

3. If a trapezium have its opposite angles supplements to each other, and have four sides 24, 26, 28, 30; required its area.

By Rule II. the area is 723·989.

4. How many square yards of paving are in the trapezium, the diagonal of which is 65 feet, and the two perpendiculars let fall on it 28 and 38·5 feet? *Ans.* 222$\frac{1}{18}$ *yards.*

5. What is the area of a trapezium, the south side being 27·40 chains, east side 35·75 chains, north side 37·55 chains, west side 41·05 chains, and the diagonal from south-west to north-east 48·35 chains? *Ans.* 123*a.* 0*r.* 11.8656*p.*

6. What is the area of a trapezium, the diagonal of which is 108½ feet, and the perpendiculars 65¼ and 60¾ feet.

Ans. 705½ *square yards.*

7. What is the area of a trapezium, the four sides being 12, 13, 14, 15? having its opposite angles supplemental.

Ans. 180·997.

8. In the four sided field A B C D, on account of obstructions in the two sides A B, C D, and in the perpendiculars B F, D E, the following measures only could be taken: namely, the two sides B C 265 and A D 220 yards, the diagonal A C 378 yards,

and the two distances of the perpendiculars from the ends of the diagonal, namely A E 100, and C F 70 yards: required the area in acres, when 4840 square yards make an acre.

Ans. 17*a.* 2*r.* 21*p.*

9. When AB = 314, BC = 232, CD = $228\frac{1}{2}$, DA = $266\frac{1}{7}$, and the diagonal A C = $417\frac{1}{2}$ feet; required the area in square yards. *Ans.* $8641\frac{1}{3}$ *square yards.*

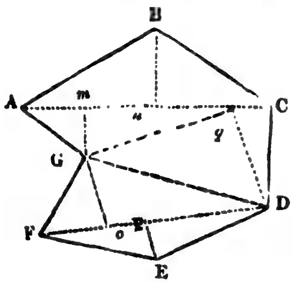
Problem V.

To find the area of an irregular polygon.

Rule.—Draw diagonals dividing the figure into trapeziums and triangles; then find the areas of all these separately, and add them together for the content of the whole figure.

Examples.

1. To find the content of the irregular figure ABCDEFGA, in which are given the following diagonals and perpendiculars: namely,



A C	5·5
F D	5·2
G C	4·4
G m	1·3
B n	1·8
G o	1·2
E p	0·8
D q	2·3

1st, For trapez. ABCG.	2nd, For trapez. GDEF.	3rd, For triangle GCD.
1·3	1·2	4·4
1·8	0·8	2·3
3·1	2·0	132
5·5	5·2	88
1·55	10·4	10·12
15·5		
17·05		

17·05 double A B C G
10·40 double G D E F
10·12 double G C D

2)37·57 double the whole.

18·785 *Ans.*

2. Required the area of the figure A B C D E F G, when A C $= 12$, F D $= 11$, G C $= 9\frac{1}{3}$, G $m = 3\frac{1}{4}$, B $n = 4$, G $o = 2\frac{1}{3}$, E $p = 1\frac{1}{3}$, and D $q = 4\frac{1}{3}$ feet.

Problem VI.

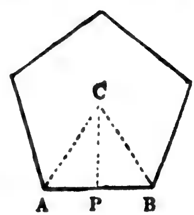
To find the area of a regular polygon.

Rule I.—Multiply the sum of the sides or perimeter of the polygon by half the perpendicular from its centre to one of its sides, and the product will be the area.

Rule II.—Multiply the square of the side of the polygon by the number opposite its name, in the column headed "Areas," in the Table to Prob. IV., Part II. and the product will be the area.

FORMULÆ.

Let $s =$ A B $=$ side of the polygon, $p =$ C P perpendicular from the centre on A B, $n =$ number of sides of the polygon, and $a =$ its tabular area; then
$A = \frac{1}{2} n p s$, and $A = a s^2$. Also
$s = \sqrt{\frac{A}{a}} = \frac{2A}{np}$, and $p = \frac{2A}{ns}$.



EXAMPLES.

1. Required the area of a regular pentagon, the side A B of of which is 25 feet, and the perpendicular C P $= 17 \cdot 205$.

By Rule I.

```
         17·205
25 × 5 = 125 = perim.
        -------
         86025
        34410
       17205
       --------
     2)2150·625
       --------
       1075·3125 sq. feet.
```

By Rule II.

```
 1·7205 table area.
    625 = 25²
 -------
  86025
 34410
103230
---------
1075·3125 sq. feet. Ans.
```

2. To find the area of the hexagon, the side of which is 20 feet. *Ans.* 1039·23 *square feet.*

3. To find the area of the trigon, or equilateral train, the side of which is 20 feet. *Ans.* 173·205 *square feet.*

4. Required the area of an octagon, the side of which is 20 feet. *Ans.* 1931·37 *square feet.*

5. What is the area of a decagon, the side of which is 20 feet. *Ans.* 3077·68 *square feet.*

6. Required the side of a decagon, the area of which is 16 square feet.

By the third formula, the side $s = \sqrt{\frac{A}{a}}$, that is,

$$\sqrt{\frac{16}{7{\cdot}6942}} = 1{\cdot}442\ feet = 1\ foot\ 5{\cdot}3\ inches.\ \ Ans.$$

7. The fence of an octagonal inclosure, within a square in a city, cost £840 at 4*s.* 8*d.* per foot; what will be the cost of the gravelling the surface at 10½*d.* per square yard?
Ans. £132 0*s.* 6½*d.*

8. The corners of a square are cut off so as to form an octagon; required the area of the octagon, the side of the square being 200 feet. *Ans.* 3681·8 *square yards.*

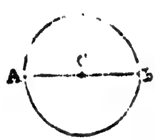
Problem VII.

To find the area of a circle when the radius, or half diameter, is given.

Rule I.—Multiply the square of the radius by 3·1416 for the area.

To find the area of a circle when the circumference is given.

Rule II.—Multiply the square of the circumference by ·07958.



Put the radius A C = r, the circumference = c, and 3·1416 = π; then

$$A = \pi r^2, \text{ and } r = \sqrt{\frac{A}{\pi}}; \text{ also}$$

$$A = \frac{c^2}{4\pi} = \tfrac{1}{2} r c, \text{ and } c = \sqrt{4 A \pi},$$

EXAMPLES.

1. Required the area of a circle, the radius of which is 5 feet.

By Rule I., or the first formula.

$$3{\cdot}1416 \times 5^2 = 3{\cdot}1416 \times 25 = 78{\cdot}54 \text{ square feet.}$$

2. The circumference of a circle is 18·4 feet, what is its area? *Ans.* 26·92 *square feet.*

3. A circular pleasure ground is to be laid out to contain

exactly an acre, required the length of the cord with which the circle must be traced.

By the second formula, the length of the chord, or

$$r = \sqrt{\frac{4840}{3 \cdot 1416}} = 39\tfrac{1}{4} \text{ yards, very nearly.}$$

4. How many square yards are in a circle whose diameter is $3\frac{1}{2}$ feet? *Ans.* 1·069.

5. How many square feet does a circle contain, the circumference being 10·9956 yards. *Ans.* 86·19543.

6. The area of the piston of a steam engine is required to be 1192 square inches to give it the requisite power; required the interior diameter of the cylinder, and its exterior circumference the thickness of the metal being one inch.

Ans. { *Interior diameter* 39 *inches nearly.* / *Exterior circumference* 10 *feet* $8\frac{3}{4}$ *inches.* }

7. The circumference of the circular paling of a plantation was found to be $235\frac{1}{4}$ yards, what is its area.

Ans. 4400 *square yards.*

8. What is the circumference of a circle, the area of which is an acre? *Ans.* 246 *yards* 1 *foot* $10\frac{1}{2}$ *inches.*

Problem VIII.

To find the area of a sector of a circle.

Rule I.—Multiply the radius, or half the diameter, by half the arc of the sector, for the area. Or, multiply the diameter by the arc of the sector, and take $\frac{1}{4}$ of the product.

Note. The arc may be found by Prob. III.

Rule II.—As 360 is to the degrees in the arc of a sector, so is the whole area of the circle, to the area of the sector.

Note. For a semicircle take one half, for a quadrant, one quarter, &c., of the whole circle.

FORMULÆ.

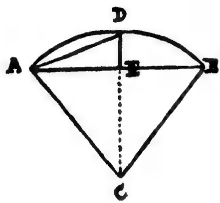
$$A = \frac{1}{2} r \times \text{arc, and } r = \frac{2A}{\text{arc}}.$$

EXAMPLE.

1. What is the area of the sector C A D B, the radius being 10, and the chord A B 16?

By Rule I. .... $100 = AC^2$
$64 = AE^2$

$36(6 = CE$
$10 = CD$



$4 = DE$

$16 = D$
$64 = A$

$80 (8{\cdot}9442719 = AD$
8

$71{\cdot}5541752$
16

$3 \,)\, 55{\cdot}5541752$
$2 \,)\, 18{\cdot}5180584$ arc A D B
$9{\cdot}2590297$ = half arc
10 = radius

$92{\cdot}590297$ *Ans.*

2. Required the area of a sector, the arc of which contains 96 degrees, the diameter being 3 feet.

$$\cdot 7854 = \tfrac{1}{4}\pi$$
$$9 = 3^2$$

$7{\cdot}0686$ = area of the whole circle.

Then by Rule II.,

as $360^\circ : 96^\circ :: 7{\cdot}0686$

or, as $30^\circ : 8^\circ :: 7{\cdot}0686 : 1{\cdot}88496$ *square feet. Ans.*

3. What is the area of a sector, the radius of which is 10 feet, and the arc 20? *Ans.* $11\frac{1}{9}$ *square yards.*

4. Required the area of a sector, the radius of which is 18 feet, and the chord of its arc 12? *Ans.* $110\frac{1}{10}$ *square feet.*

5. How may square yards are in a sector of 187° 37′, the radius of the circle being 289? *Ans.* 15194 *square yards.*

6. Required the area of a sector, the radius of which is 25 feet, and its arc contains 147° 29′.

Ans. 804·4 *square feet nearly.*

7. What is the area of a sector, the chord of the arc of which is 24 feet, and its height 6? *Ans.* 208·572 *square feet.*

8. Required the area of a sector greater than a semicircle, the chord of its arc being 12, and its diameter 15 feet.

Ans. 124½ *square feet.*

PROBLEM IX.

To find the area of a segment of a circle.

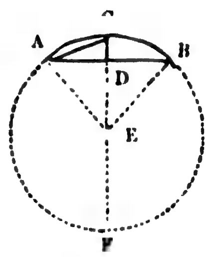
RULE I.—Find the area of a sector having the same arc as the segment, by the last problem; find also the area of the triangle, formed by the chord of the segment and the two radii of the sector: then add these two together for the area, when the arc is greater than a semicircle; but subtract for the area, when the arc is less that a semicircle.

RULE II. Divide the height or versed sine of the segment by the diameter, and find the quotient in the column of versed sines, in Table I., at the end of the book. Take out the corresponding area, in the next column on the right hand, and multiply by the square of the diameter for the area.

FORMULÆ.

Put r = A E, C = A B, r = C D, p = E D, t = tabular area, and a = arc A C B; then

$$A = \tfrac{1}{2}(ar - Cp) = (2r)^2 t = \tfrac{2}{3} r\sqrt{C^2 + \tfrac{3}{5}r^2}$$



NOTE 1. When the segment is greater than a semicircle, find the area of the remaining segment, and subtract from the whole area of the circle for the required area.

NOTE 2. The first rule or formula gives an appoximate value of the area, not very far from the truth; the last formula is still nearer the truth; and the second rule or formula may be considered as exactly true.

EXAMPLES.

1. Required the area of the segment A C B D A, its chord A B being 12, and the radius A E or C E 10 feet.

First find C D and A C from the properties of the figure, and the length of the arc A C B by Prob. VII., Part II.; then find the area by Rule I.; thus $DE = \sqrt{AE^2 - AD^2} = \sqrt{10^2 - 6^2} = 8$, $CD = CE - DE = 10 - 8 = 2$, and $AC = \sqrt{AD^2 + CD^2} = \sqrt{\ \ \ \ +2^2} = 6{\cdot}324555$; whence $\frac{6{\cdot}324555 \times 8 - 12}{2} =$ $38{\cdot}59644$ = arc A C B, and by Rule I., $\tfrac{1}{2}(38{\cdot}59644 \times 10) - \tfrac{1}{2}(12 \times 8) = 16{\cdot}3274$ *square feet. Ans.*

By Rule II. The example being the same as before, we have C D equal to 2; and the diameter 20.

Then 20) 2 (·1
And to ·1 answers ...... .040875 per Table 1.
Square of diameter...... 400

Ans. 16·3500 *square feet.*

By the third formula, the same example being still used, $A = \frac{2}{3} v \sqrt{C^2 + \frac{8}{5} v^2} = \frac{4}{3} \sqrt{12^2 + \frac{8}{5} 2^2} = 16{\cdot}3511$ square feet, which is very near the truth.

2. What is the area of the segment, the height of which is 2, and the chord 20 feet. *Ans.* 26·36046.

3 What is the area of the segment, the height of which is 18, and diameter of the circle 50 feet? *Ans.* 636·375.

4. Required the area of the segment, the chord of which is 16, the diameter being 20 feet. *Ans.* 44·7292.

5. What is the area of a segment, the arc of which is a sextant, the whole circumference of the circle being 25 feet? *Ans.* 1·4312 *square feet.*

6. The chord of a segment is 40, and its height 8 feet; what is its area? *Ans.* 220 *square feet nearly.*

PROBLEM IX.

To find the area of a circular zone.
(*See figure to* Prob. VIII., Part II.)

RULE.—The zone being first divided into a trapezoid (ABCD) and two equal segments (BHD and AC), find the area of the trapezoid by Prob. III., and the areas of the two segments by Prob. IX.; which areas, being added together, will give the area of the zone.

EXAMPLES.

1. The breadth of a zone is 42 feet, and its parallel chords are 48 and 36 feet, required the area.
Ans. 253·08 *square yards.*

2. The two parallel chords of a circular zone are each 100 yards, and the radius of the circle 72 yards; required the area of the zone. *Ans.* 13508¼ *square yards.*

3. The parallel chords of a circular zone are each 2¼ feet, and the radius of the circle 1¾; required the area.
Ans. 6¼ *square feet nearly.*

PROBLEM X.

To find the area of a circular ring, or space included between two concentric circles.

Take the difference between the two circles, for the ring; or multiply the sum of the radii by their difference, and multiply the product by 3·1416 for the answer.

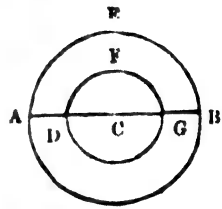
FORMULÆ.

$A = \pi (R^2 - r^2) = \frac{1}{4\pi}(C^2 - c^2)$; in which R and r are the greater and lesser radii, and C and c the greater and lesser circumferences.

EXAMPLES.

1. The diameters of the two concentric circles being A B 20 and D G 12 feet, required the area of the ring contained between their circumferences A E B A, and D F G D.

A C =	10	3·1416
D C =	6	64
sum	16	12·5664
dif.	4	188·496
	64	201·0624



2. The diameters of two concentric circles being 20 and 10 feet; required the area of the ring between their circumferences. *Ans.* 235·72 *square feet.*

3. What is the area of a ring, the diameters of its bounding circles being 6 and 4 feet? *Ans.* 15·708.

4. The circular fences on each side a gravel walk, surrounding a shrubbery, are 800 and 714 feet in length; what is the area of the walk, and what did it cost laying with gravel at 1*s.* 4½*d.* per square yard. *Ans.* { *Area* 1151 *square yards.* *Cost* £79 2*s.* 7½*d.*

PROBLEM XI.

To find the area of a lune A C B D A.

RULE.—Find by Prob. VIII. the areas of the segments A C B and A D B, formed by the chord A B of the two arcs of the lune, and the difference of these areas will be the area required.

EXAMPLES.

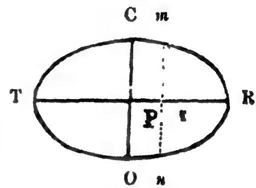
1. What is the area of lune, the chord AB of which is 24 ft. and the heights of its two arcs 5 and $3\frac{1}{4}$ ft.? *Ans.* $25\frac{3}{4}$ *sq. ft.*

2. The chord of a lune is 40 feet, and the heights of its arcs 4 and 20 feet; required the area. *Ans.* 57·867 *square yards.*

Problem XII.

To find the area of an ellipse.

Rule.—Multiply the product of the semiaxes T P, C P by 3·1416 for the area.



FORMULA.

$A = a\,b\,\pi$, in which a and b are the semiaxes.

EXAMPLES.

1. The axes of an elliptical shrubbery in a park are 300 and 200 feet; required the area.

Ans. 5236 *square yards,* = 1 *acre* 396 *square yards.*

2. Required the area of an ellipse, the axes of which are 70 and 50 yards. *Ans.* 2748 *square yards* 8 *feet.*

Problem XIII.

To find the area of an elliptical segment, the chord of which is parallel to one of the axes. (*See last figure.*)

Rule.—Divide the height of the segment by that axis of the ellipse of which it is a part; and find in the table of circular segments at the end of the book, a circular segment having the same versed sine as this quotient. Then multiply continually together, this segment, and the two axes, for the area required.

EXAMPLES.

1. What is the area of an elliptic segment m R n, whose height R r is 20; the tranverse T R being 70, and the conjugate C O 50 feet?

70) 20 (·285$\frac{5}{7}$ the tabular versed sine.
The corresponding segment
is ·185166
70

12·961620
50

648·081000 square feet, the area required.

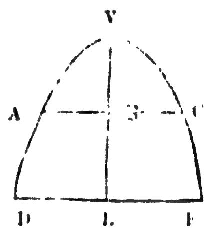
2. What is the area of an elliptic segment, cut off parallel to the shorter axis, the height being 10, and the axes 25 and 35 feet? *Ans.* 162·021 *square feet.*

3. What is the area of the elliptic segment, cut off parallel to the longer axis, the height being 5, and the axes 25 and 35 feet? *Ans.* 97·8458 *square feet.*

PROBLEM XIV.

To find the area of a parabola.

RULE.—Multiply the axis or height V E by the base or double ordinate D F, and $\frac{2}{3}$ of the product will be the area.



FORMULA.

$A = \frac{2}{3}\, a\, d$, in which a is the axis, and d the double ordinate.

EXAMPLES.

1. Required the area of the parabola A V C, the axis V B being 2, and the double ordinate A C 12 feet.

$\frac{2}{3} \times 12 \times 2 = 16$ square feet, the area required.

2. The double ordinate of a parabola is 20 feet, and its axis or height 18: required the area of the parabola. *Ans.* 240 *square feet.*

PROBLEM XV.

To find the area of a parabolic frustrum A C F D.

Cube each end of the frustrum, and subtract the one cube from the other; then multiply that difference by double the altitude, and divide the product by triple the difference of their squares, for the area.

$A = \frac{2}{3}\, a . \frac{C^3 - c^3}{C^2 - c^2}$, in which a is the altitude, and C and c the parallel chords.

EXAMPLES.

1. Required the area of the parabolic frustrum A C F D, A C being 6, D F 10, and the altitude B E 4 feet.

Ends.	Squares.	Cubes.
D F = 10	100	1000
A C = 6	36	216
	64 dif.	784
	3	8 = 2 B E
	192)	6272 ($32\frac{128}{192} = 32\frac{2}{3}$ *Ans.*
		576
		512
		384
		128

2. What is the area of the parabolic frustrum, the two ends of which are 6 and 10, and its altitude 3 feet. *Ans.* $24\frac{1}{2}$ *square feet.*

NOTE. Those who wish for further information on the areas of the conic sections, are referred to the works of *Emerson, Hamilton*, &c., it being foreign to the object of this work to give more on this subject.

PROBLEM XVI.

To find the areas of irregular figures whether bounded by straight lines or curves.

CASE I.—*When the figure is long and narrow.*

RULE.—Take the perpendicular breadth at several places, at equal distances; to half the sum of the first and last two breadths, add the sum of all the intermediate breadths, and multiply the result by the common distance between the breadths for the area.

CASE II.—*When the breadths or perpendiculars are taken at unequal distances, the figure being long and narrow.*

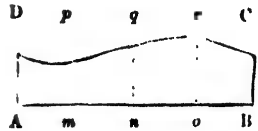
RULE I.—Find the areas of all the trapezoids and triangles separately, and add them together for the area.

RULE II.—Add all the breadths together, and divide the sum by the whole number of them for the mean breadth, which multiply by the length for the area.—This method is not very correct, but may do where great accuracy is not required.

EXAMPLES.

1. The perpendicular breadths, or offsets of an irregular figure at five equidistant places are A D = 8·2, $m\,p$ = 7·4, $n\,q$ = 9·2, $o\,r$ = 10·2, B C = 8·6 feet; and the common distances A m = $m\,n$ = &c. = 50 feet; required the area.

By Rule I, Case I.



$$
\begin{array}{r}
8{\cdot}2 \\
8{\cdot}6 \\
\hline
2)16{\cdot}8 = \text{sum} \\
\hline
8{\cdot}4 = \tfrac{1}{2}\text{ sum} \\
7{\cdot}4 \\
9{\cdot}2 \\
10{\cdot}2 \\
\hline
35{\cdot}2 \\
50 \\
\hline
\end{array}
$$

Ans. 1760·0 *square feet.*

2. The length of an irregular plank is 25 feet, and its perpendicular breadth at six equidistant places are 17·4, 20·6, 14·2, 16·5, 20·1, and 24·4 inches; required the area.

Ans. $30\frac{1}{8}$ *square feet.*

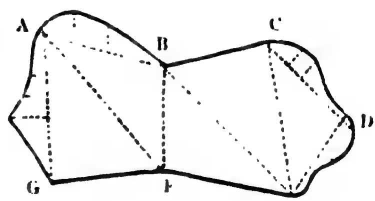
3. Take the dimensions and find the area of the annexed irregular figure, by Rule I. and II., Case II.

CASE III. *When the breadth of the figure is large and its boundary curved or crooked.*

RULE.—Divide the figure into trapeziums and triangles, in the most convenient manner, taking offsets to the curved or crooked portion of the boundary. Find the areas of the trapeziums, triangles, and the offset pieces separately, which, being added together, will give the required area of the whole figure.

EXAMPLE.

The annexed figure is divided into two trapeziums A B F G, B C E F, and one triangle C D E, with offsets on A B, A G, C D, and D E. It is required to measure the several parts of the figure, and to find its area.



The areas of the trapeziums are found by letting fall perpendiculars on the diagonals A F, B E by Prob. IV., and the area of the triangle by Prob. II., the areas of the several offset pieces being found by one or other of the cases of this Problem.

PROMISCUOUS EXERCISES.

1. The sides of three squares are 6, 8, and 24 feet; required the side of a square that shall have an area equal to all the three. *Ans.* 26 *feet.*

2. In cutting a circle, the largest possible, out of a card-board 5 feet square, how much will be wasted.

Ans. 5·365 *square feet.*

3. The area of a square is 72 square feet; required the length of its diagonal. *Ans.* 12 *feet.*

4. A ditch 13 yards wide surrounds a circular fortress, the circumference of the fortress being 704 yards; required the area of the ditch. *Ans.* 2 *acres nearly.*

5. What is the area of a circular table the diameter of which is 59 inches. *Ans.* 19 *square feet nearly.*

6. What is the area of an isosceles triangle, the base of which is 5 feet 10 inches, and each side $8\frac{1}{2}$ feet?

Ans. 23 *square feet* $41\frac{1}{3}$ *inches.*

7. Required the side of a decagon the area of which is 9 square feet. *Ans.* 1 *foot* 1 *inch nearly.*

8. The side of a square is 50 yards, and its corners are cut off so as to form an octagan; required the area of the octagon.

Ans. 2071 *square yards.*

PART IV.

MENSURATION OF SOLIDS.

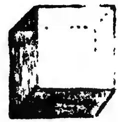
DEFINITIONS.

1. A SOLID has three dimensions, length, breadth, and thickness.

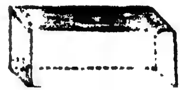
2. A prism is a solid, or body, whose ends are any plane figures, which are parallel, equal, and similar; and its sides are parallelograms.

A prism is called a triangular one when its ends are triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

3. A cube is a square prism, having six sides, which are all squares. It is like a die, having its sides perpendicular to one another.



4. A parallelopipedon is a solid having six rectangular sides, every opposite pair of which are equal and parallel.



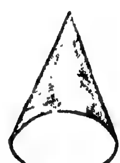
5. A cylinder is a round prism, having circles for its ends.

NOTE. A prism is called a right one, when its sides are perpendicular to its ends; and an oblique prism when its sides are inclined to its ends.

6. A pyramid is a solid having any plane figure for a base, and its sides are triangles, the vertices of which meet in a point at the top, called the vertex of the pyramid.

The pyramid takes names according to the figure of its base, like the prism; being triangular, or square, or hexagonal, &c.

7. A cone is a round pyramid, having a circular base.



8. A sphere is a solid bounded by one continued convex surface, every point of which is equally distant from a point within, called the centre.—The sphere may be conceived to be formed by the revolution of a semicircle about its diameter, which remains fixed.

9. The axis of a solid, is a line drawn from the middle of one end, to the middle of the opposite end; as between the opposite ends of a prism. Hence the axis of a pyramid, is the line from the vertex to the middle of the base, or the end on which it is supposed to stand, as O V. And the axis of a sphere, is the same as a diameter, or a line passing through the centre, and terminated by the surface on both sides.

NOTE. It is called a right pyramid when the axis is perpendicular to the base, but when inclined to the base, it is called an oblique pyramid.

10. The height or altitude of a solid, is a line drawn from its vertex or top, perpendicular to its base.—This is equal to the axis in a right prism or pyramid ; but in an oblique one, the height is the perpendicular side of a right-angled triangle, whose hypothenuse is the axis.

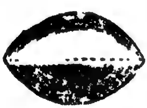
11. Also a prism or pyramid is regular or irregular, as its base is a regular or an irregular plane figure.

12. The segment of a pyramid, sphere, or any other solid, is a part cut off the top by a plane parallel to the base of that figure.

13. A frustrum or trunk, is the part that remains at the bottom, after the segment is cut off.

14. A zone of a sphere, is a part intercepted between two parallel planes. When the ends, or planes, are equally distant from the centre, on both sides, the figure is called the middle zone.

15. The sector of a sphere, is composed of a segment less than a hemisphere or half sphere, and of a cone having the same base with the segment, and its vertex in the centre of the sphere.



16. A circular spindle, is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.

17. A regular body, is a solid contained under a certain number of equal and regular plane figures of the same sort.

18. The faces of the solid are the plane figures under which it is contained ; and the linear sides, or edges of the solid, are the sides of the plane faces.

19. There are only five regular bodies : namely, 1st, the tetrahedron, which is a regular pyramid, having four triangular faces : 2nd, the hexahedron, or cube, which has 6 equal square faces : 3rd, the octahedron, which has 8 triangular faces : 4th, the dodecahedron, which has 12 pentagonal faces : 5th, the icosahedron, which has 20 triangular faces.

TABLE OF SOLID MEASURE.

1728 cubic inches	=	1 cubic foot.
27 cubic feet	=	1 cubic yard.
277·274, or 277¼ nearly } cubic inches	=	1 gallon.

PROBLEM I.

To find the solidity of a cube.

RULE.—Cube one of its sides for the content; that is, multiply the side by itself, and that product by the side again.

FORMULÆ.

Let l = length of the side of the cube, S its solidity, and s its surface; *(which two last are also used to represent the solidities and surfaces of all the solids in the following problems)* then,

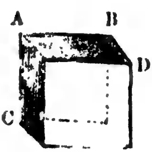
$$S = l^3, \text{ and } l = \sqrt[3]{S}. \quad \text{Also } s = 6\,l^2.$$

EXAMPLES.

1. If the side A B, or A C, or B D, of a cube be 24 inches, what is its solidity or content?

By the Rule or the first Formula.

```
  24
  24
  --
  96
 48
----
 576
  24
----
2304
1152
-----
13824 Ans.
```



2. How many solid yards are in a cube the side of which is 22 feet? *Ans.* 394 *solid yards* 10 *feet.*

3. Required how many solid feet are in the cube the side of which is 18 inches? *Ans.* $3\frac{3}{8}$.

4. What is the content of a cube, measuring 6 feet 8 inches every way? *Ans.* 296 *cubic feet* 3'. 6''. 8'''.

5. A cubical box contains 343 cubic feet; required the length of its side.

By the second formula $l = \sqrt[3]{S} = \sqrt[3]{343} = 7$ feet.

6. How may square feet of deal will make a cubical box, lid included, each side of the box being 3 feet?

By the last formula, $s = 6\,l^2 = 6 \times 3^2 = 54$ square feet.

PROBLEM II.

To find the solidity of a parallelopipedon.

RULE.—Multiply the length, breadth, and depth, or altitude.

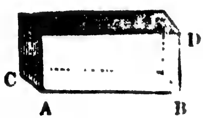
all continually together, for the solid content: that is, multiply the length by the breadth, and that product by the depth.

FORMULÆ.

Put l = length, b = breadth, and d = depth of the solid; then $S = l\,b\,d$, $l = \frac{S}{b\,d}$, $b = \frac{S}{l\,d}$, $d = \frac{S}{l\,b}$. Also, $s = 2\left\{l\,(b + d) + b\,d\right\}$.

EXAMPLES.

1. Required the content of the parallelopipedon, whose length A B is 6 feet, its breadth A C 2½ feet, and altitude B D 1¾ feet?



$$
\begin{array}{r l}
1{\cdot}75 & = \text{B D} \\
6 & = \text{A B} \\
\hline
10{\cdot}50 & \\
2{\cdot}5 & = \text{A C} \\
\hline
5250 & \\
2100 & \\
\hline
26{\cdot}250 & \textit{Ans.}
\end{array}
$$

2. Required the content of a parallelopipedon, the length of which is 10·5, breadth 4·2, and height 3·4. *Ans.* 149·94.

3. How many cubic feet are in a block of marble, the length of which is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches? *Ans.* 21⅑.

4. A stone in the ruins of the walls of Baalbec is 36 feet in length, 14 in breadth, and 12 in thickness; required its content, and its weight at the rate of 180 lbs. per cubic foot.

Ans. 11088 *cubic feet, and weight* 891 *tons.*

5. A rectangular cistern is to be made 32 feet in length and 12 in breadth, and to hold 1920 cubic feet of water; what must be its depth?

By the third formula the depth $d = \frac{S}{l\,b} = \frac{1920}{32 \times 12} = 5$ feet.

6. What quantity of deal is there in a box 3½ feet long, 2 wide, and 1½ deep?

By the last formula $s = 2\left\{3\tfrac{1}{2}\,(2 + 1\tfrac{1}{2}) + 2 \times 1\tfrac{1}{2}\right\} = 30\tfrac{1}{2}$ square feet.

PROBLEM III.

To find the solidity of any prism or cylinder.

RULE.—Find the area of the base or end; which multiply by the height or length; and the product will be the content.

To find the area of the surface of a prism or cylinder.

RULE.—Multiply the circumference of the base or end by the length or height, and the product will be the area required.

NOTE. If the *whole* surface be required, the area of the two ends must be added to the area found by the rule.

FORMULÆ.

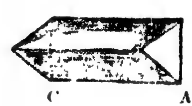
Put $l =$ length or height, as before; a the area, and c the circumference of the base; then

$S = a\,l = \frac{c^2\,l}{4\,\pi}$, $l = \frac{S}{a}$. Also $s = c\,l + 2\,a =$ surface of the prism, including the two ends, and $s = \pi\,r\,l =$ convex surface of the cylinder, exclusive of the ends, r being the radius of the base.

EXAMPLES.

1. Required the content of a triangular prism, the length A C of which is 12 feet, and each side of its equilateral base $2\frac{1}{2}$ feet.

By the Rule or first Formula.



$$
\begin{array}{rrl}
 & \cdot433013 & \text{tabular No.} \\
 & 6\frac{1}{4} & = (2\frac{1}{2})^2 \\
\hline
 & 2.598078 & \\
 & 108253 & \\
\hline
a = & 2\cdot706331 & \text{area of end} \\
l = & 12 & \text{length} \\
\hline
\end{array}
$$

Ans. 32·475972 *solid feet.*

2. Required the solidity of a triangular prism, the length of which is 10 feet, and the three sides of its triangular end or base, are 5, 4, 3 feet? *Ans.* 60 *cubic feet.*

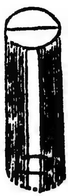
3. What is the content of a hexagonal prism, the length being 8 feet, and each side of its end 1 foot 6 inches.

Ans. 46·765 *cubic feet.*

4. Required the content of a cylinder, the length of which is 20 feet, and circumference $5\frac{1}{2}$ feet.

By the second formula,

$$S = \frac{c^2\,l}{4\,\pi} = (5\tfrac{1}{2})^2 \times 20 \times \cdot07958 = 48\cdot146 \text{ cubic feet.}$$



5. What is the convex surface of a cylinder, the length of which is 16 feet, and its diameter 2 feet 3 inches?

By the last formula,

$s = \pi r l = 3{\cdot}1416 \times 2\frac{1}{4} \times 16 = 113{\cdot}0976$ sq. feet.

6. Required the *whole* superficial area of a cylinder, the length of which is 15 feet, and diameter $5\frac{1}{3}$ feet?

Ans. $32\frac{7}{8}$ *square yards.*

7. The *whole* superficial area of a triangular prism is 143 square feet, and each side of its equilateral ends 5 feet; required its length?

By transposing the third formula,

$$l = \frac{s - 2a}{c} = 6 \text{ feet } 8 \text{ inches nearly.}$$

8. The diameter of a cylinder is 12 feet, and its length 20; required the content?

Ans. 2262 *cubic feet nearly.*

9. How many cubic feet of stone is there in a round pillar, the height of which is 16 feet, and diameter 2 feet 3 in.? *Ans.* 63·62 *cubic ft.*

10. How many square yards of painting are there in the convex surface of a column, the length of which is 20 feet, and its diameter 2 feet?

Ans. 13 *square yards* $8\frac{2}{3}$ *feet nearly.*

Problem IV.

To find the solidity of any cone, or any pyramid.

Rule.—Compute the area of the base, then multiply that area by the height, and take $\frac{1}{3}$ of the product for the content.

To find the convex surface of a right cone, or the slant surface of a right pyramid.

Rule.—Multiply the circumference of the base by the slant height, or length of the side, and take half the product for the surface.

FORMULÆ.

$S = \frac{1}{3} a\, l,\ a = \frac{3\,S}{l},\ l = \frac{3\,S}{a}$. Also $s = \frac{1}{2} c\, l'$, l' being the slant height. When the whole surface is required, the area of the base must be added.

EXAMPLES.

1. What is the solidity of a cone, the height C D of which is $12\frac{1}{2}$ feet, and the diameter A B of the base $2\frac{1}{2}$?

Here $2\frac{1}{2} \times 2\frac{1}{2} \times = \frac{5}{2} \times \frac{5}{2} \times \frac{25}{4} = 6\frac{1}{4} = AB^2$.

Then ·7854
$6\frac{1}{4}$
———
4·7124
19635
———
4·90875 area of base
$12\frac{1}{2}$ height C D
———
58·90500
2·454375
———
3) 61·359375
20·453125 *Ans.*

2. What is the solid content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?

1·720477 tab. area
4 square side
———
6·881908 area base
4 $\frac{1}{3}$ of height C O
———
Ans. 27·527632 *cubic feet.*

3. What is the content of a cone, its height being $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22·561 *cubic feet.*

4. Required the content of a triangular pyramid, its height being 14 feet 6 inches, and the three sides of its base, 5, 6, 7.

Ans. 71·0351 *cubic feet.*

5. What is the content of a hexagonal pyramid, the height of which is 6·4, and each side of its base 6 inches.

Ans. 1·38 *cubic feet.*

6. If the diameter of the base A B be 5 feet, and the side of the cone A C 18, required the convex surface.

3·1416
5 diameter
———
15·7080 circumference
18
———
125664
15708
———
2) 282·744
Ans. 141·372 *square feet.*

7. What is the convex surface of a cone, the slant side of which is 20, and the circumference of its base 9 feet?

Ans. 90 *square feet.*

8. Required the convex surface of a cone, the slant height of which is 50 feet, and the diameter of its base 8 feet 6 inches?

Ans. 667·59 *square feet.*

9. The side of the equilateral base of a triangular pyramid is 5 feet, and its solid content $62\frac{1}{2}$ cubic feet; required its perpendicular height.

Ans. By the third formula, the height is found 17 *feet* 4 *inches nearly.*

10. Required the weight of a hexagonal pyramid of marble, each side of the base of which is 1 foot 3 inches, and the vertical height 10 feet, the weight of the marble being 170 lbs. per cubic foot. *Ans.* 1 *ton.* 0 *cwt.* $18\frac{1}{2}$ *lbs.*

11. A cone contains 8 solid feet, and its height is 2 feet: what is the circumference of its base? *Ans.* 12·28 *ft. nearly.*

12. The circumference of the base of a cone is 33 feet, and the slant height 8 feet 9 inches; required the content?

Ans. 202·65 *cubic feet.*

Problem V.

To find the solidity of the frustrum of a cone, or any pyramid.

General Rule.—To the areas of the two ends add the square root of their product, and multiply the sum by $\frac{1}{3}$ of the height for the solidity.

FORMULÆ.

If A and a be the areas of the greater and lesser ends; then,

$$S = \tfrac{1}{3}\,(A + a + \sqrt{A\,a})\,l.$$

When the solid is the frustrum of a cone, or of a pyramid, having its ends regular polygons.

Rule.—To the sum of the squares of the radii of the ends, if a cone, or of the sides of the ends, if a pyramid, add their product; and multiply the sum by 3·1416, if a cone, or by the tabular number of the polygon, if a pyramid, and again by $\frac{1}{3}$ of the height for the content.

FORMULÆ.

$$S = \tfrac{1}{3}\,(R^2 + r^2 + R\,r)\,l\,\pi,$$

in which R and r are the radii of the ends, if a cone, or the sides of the ends, if a pyramid. In the latter case π represents

the tabular number of the polygon. If R and r be taken as the circumferences of the ends of a cone, then π must be taken $= \cdot 07958$.

To find the convex surface or frustrum of a cone, or the slant surface of a pyramid.

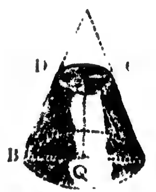
RULE.—Multiply the sum of the circumferences of the two ends by ½ the slant height of the frustrum for the required surface.

NOTE. When the *whole* surface is required the areas of the two ends must be added to the result of the Rule.

EXAMPLES.

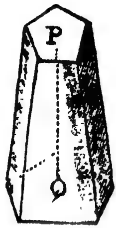
1. What is the content of a frustrum of a cone, the height of which is 20 inches, and the diameters of its two ends 28 and 20 inches?

28	28	20
28	20	20
224	560	400
56	784	
784	400	
	1744	
	·2618	
	13952	
	1744	
	10464	
	3488	
	456·5792	
	20 = P Q	



Ans. 9131·5840 *solid inches.*

2. Required the content of a pentagonal frustrum, the height of which is 5 feet, each side of the base 1 foot 6 inches, and each side of the less end 6 inches.



18	18	6
18	6	6
144	108	36
18	324	
324	36	

3) 468

156 $\frac{1}{3}$ of sum
1·720477 tab. area

10322862
8602385
1720477

268·394412 mean area
5 height P Q

144 { 12 | 1341·972060
{ 12 | 111·831005
9·319250 *Ans. in cubic feet.*

3. What is the solidity of the frustrum of a cone, the altitude being 25, the circumference at the greater end 20, and at the less end 10 feet? *Ans.* 464·205 *cubic feet.*

4. How many solid feet are in a piece of timber, whose bases are squares, each side of the greater end being 15 inches, and each side of the less end 6 inches; also the length, or perpendicular altitude is 24 feet? *Ans.* 19½ *cubic feet.*

5. To find the content of the frustrum of a cone, the altitude being 18, the greatest diameter 8, and the least 4 feet? *Ans.* 527·7888 *cubic feet.*

6. What is the solidity of a hexagonal frustrum, the height being 6 feet, the side of the greater end 18 inches, and of the less 12 inches? *Ans.* 24·68 *cubic feet.*

7. The girts of the trunk of a tree at its two ends are 15 and 10 feet, and its length 48 feet; how many solid feet does it contain? (R and r being taken for the girts in the second formula, &c.) *Ans.* 604¾ *nearly.*

8. The height of the frustrum of an octagonal pyramid is 48 feet, and the sides of its ends 26 and 19; required the solid content. *Ans.* 118279 *cubic feet.*

9. The sides of the ends of the frustrum of a square pyramid are 6 and 4 feet, and its slant length 20 feet, required its slant surface,

$$\left.\begin{array}{l}6 \times 4 = 24 \\ 4 \times 4 = 16\end{array}\right\} \text{circumf. of ends.}$$

$$\begin{array}{r}\overline{40} \text{ sum} \\ 10 = \tfrac{1}{2} \text{ length} \\ \hline 9)\ 400 \\ \hline 44\tfrac{4}{9} \textit{ square yards}\end{array}$$

NOTE. The slant length is measured from the middle of one side to that of its corresponding side.

10. The slant height of tower, in the form of a hexagonal pyramid, is 74 feet, each side of the base $7\frac{1}{2}$, each side of the top $2\frac{1}{2}$ feet; required the area of the sides, and the expense of painting it at 1*s.* 3*d.* per square yard.
Ans. 2220 *square feet, and* £15 8*s.* 4*d.*

11. What is the convex surface of the frustrum of a cone, the slant height of the frustrum being 12·5, and the circumferences of the two ends 6 and 8·4 feet? *Ans.* 90 *square feet.*

12. Required the convex surface of the frustrum of a cone, the side of the frustrum being 10 feet 6 inches, and the circumferences of the two ends 2 feet 3 inches, and 5 feet 4 inches?
Ans. $39\frac{1}{16}$ *square feet.*

13. The *perpendicular* height of the frustrum of a cone is 3 feet, and the circumferences of the base and top 9 and 6 feet; required the *whole* surface? *Ans.* 68·35 *square feet.*

PROBLEM VI.

To find the solidity of a wedge.

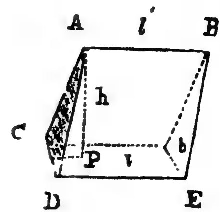
RULE.—To the length of the edge add twice the length of the back or base, and reserve the sum; multiply the height of the wedge by the breadth of the base; then multiply this product by the reserved sum, and take $\frac{1}{6}$ of the last product for the content.

FORMULA.

$S = \frac{1}{6}(2\,l + l')\,b\,h$, the symbols denoting the parts shewn on the following figure.

EXAMPLES.

1. What is the content in feet of a wedge, the altitude A P of which is 14 inches, its edge A B 21 inches, and the length of its base D E 32 inches, and its breadth C D $4\frac{1}{2}$ inches?




```
21        14
32        4½
32        ——
——        56
85         7
——        ——
          63
          85
        ————
         315
        504
        —————
      6 | 5355
1728 {12 |  892·5 Ans. in cubic inches
     {12 |   74·375
     {12 |    6·197916
         |     ·516493 Ans. in cubic feet, or little more than
        ——————                          half a cubic foot.
```

2. The edge and base of a wedge are respectively 9 feet, and 5 feet 4 inches in length, the base is 2 feet 8 inches in breadth, and the height 3 feet 6 in.; required the content of the wedge.

Ans. 30 *cubic feet*, 7'. 1''. 4'''.

3. The height and length of edge, the length and breadth of base of a wedge are each 2 feet: what is its solidity?

Ans. 4 *cubic feet.*

Problem VII.

To find the solidity of a prismoid.

Definition.—The ends of a prismoid are parallel and dissimilar rectangles or trapezoids; the solid is, therefore, the frustrum of a wedge, the part of the wedge next the edge being cut off.

Rule.—Add into one sum, the areas of the two ends and 4 times the middle section parallel to them, and $\frac{1}{6}$ of that sum will be a mean area; which being multiplied by the height will give the content.

Note. For the length of the middle section, take half the sum of the lengths of the two ends; and for its breadth, take half the sum of the breadths of the two ends.

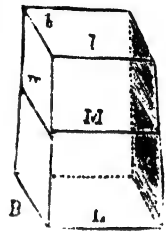
FORMULA.

$S = \frac{1}{6}(L\,B + l\,b + 4\,M\,m)$, the symbols representing the parts shewn in the following figure.

EXAMPLE.

1. How many cubic feet are there in a stone, the ends of

which are rectangles, the length and breadth of the one being 14 and 12 inches; and the corresponding sides of the other 6 and 4 inches: the perpendicular height being $30\frac{1}{2}$ feet?



$$\begin{array}{rrr}
14 & 10 & 6 \\
12 & 8 & 4 \\
\hline
168 & 80 & 24 \\
 & 4 & \\
\hline
 & 320 & \\
 & 168 & \\
 & 24 & \\
\hline
5) & 512 & \\
\hline
 & 85\frac{1}{3} & \text{mean area in inches} \\
 & 30\frac{1}{2} & \text{height} \\
\hline
 & 2560 & \\
 & 42\frac{2}{3} & \\
\hline
144\left\{\begin{array}{l}12\\12\end{array}\right. & 2602{\cdot}\dot{6} & \\
 & 216{\cdot}\dot{8} & \\
 & 18{\cdot}0\dot{7}\dot{4} & \textit{Ans.}
\end{array}$$

2. What is the content of a railway coal waggon, of which the length and breadth at top are $81\frac{1}{2}$ and 55 inches at bottom, the length and breadth are 41 and $29\frac{1}{2}$ inches, and the height $47\frac{1}{2}$ inches? *Ans.* $73\frac{1}{6}$ *cubic feet.*

NOTE. Several railway cuttings are in the form of a prismoid, with dissimilar trapezoidal ends. The following is an example of this kind, the bottom width being the same throughout.

3. The top widths of a railway cutting are 120 and 90 feet, their respective depths 30 and 20 feet, the bottom width 30 feet, and the length of the cutting 3 chains or 66 yards; required the content in cubic yards. *Ans.* $12466\frac{2}{3}$ *cubic yards.*

PROBLEM VIII.

To find the solidity of a sphere or globe.

RULE.—Multiply the cube of the diameter by ·5236.

FORMULÆ.

$$S = \tfrac{1}{6} D^3 \pi, \text{ and } D = \sqrt[3]{\frac{6S}{\pi}}.$$

EXAMPLES.

1. The diameter of a sphere is 12 feet; required its solidity?

$$12^3 \times \cdot5236 = 904\cdot7808 \text{ cubic feet.}$$

2. Find the content and weight of an ivory ball $3\frac{1}{2}$ inches in diameter, the weight of ivory being 1820 ounces (Av.) per cubic foot.

Ans. Content 22·448 *cubic in.*, and weight 24 *ounces nearly.*

3. A $2\frac{1}{2}$ inch cube of ivory is turned into a sphere of the same diameter; what weight of ivory will be lost?

Ans. 7·68 *ounces.*

4. Required the solid content of the earth, supposing its circumference to be 25000 miles?

Ans. 263858149120 *cubic miles.*

PROBLEM IX.

To find the solidity of a spherical segment.

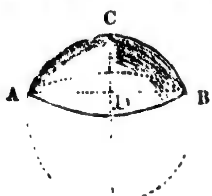
RULE.—To three times the square of the radius of its base, add the square of its height; then multiply the sum by the height, and the product again by ·5236.

FORMULA.

$$S = \tfrac{1}{6}(3r^2 + h^2)\,h\,\pi, \text{ in which } r = AB, \text{ and } h = CD.$$

EXAMPLES.

1. Required the content of a spherical segment, its height being 4 inches, and the radius of its base 8?



8	4	·5236
8	4	832
64	16	10472
3	192	15708
		41888
192	208	435·6352 *Ans.*
	4	
	832	

2. What is the solidity of the segment of a sphere, the height of which is 9, and the diameter of its base 20 feet?

Ans. 1795·4244 *cubic feet.*

3. Required the content of the spherical segment, the height of which is $2\frac{1}{4}$, and the diameter of its base 8 61684 feet?

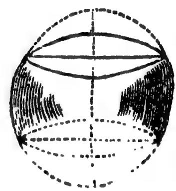
Ans. 71 5695 *cubic feet.*

FORMULA.

$$S = \tfrac{1}{6}\{3(R^2 + r^2) + h^2\}h\pi.$$

EXAMPLES.

1. What is the solid content of a zone, its greater diameter being 12 inches, the lesser 8, and the height 10 inches?



$$\begin{aligned} 6^2 &= 36 \\ 4^2 &= 16 \\ \tfrac{1}{3} \times 10^2 &= 33\tfrac{1}{3} \\ \hline & 85\tfrac{1}{3} \end{aligned}$$

$85\frac{1}{3} \times 10 \times 1{\cdot}5708 = 1340{\cdot}416$ *cubic in.*, the content required.

2. Required the content of a zone, the greater diameter is 12, less diameter 10, and height 2 feet. *Ans.* 195·8264 *cubic ft.*

3. What is the content of a middle zone, the height being 8 feet, and the diameter of each end 6 feet?

Ans. 494·2784 *cubic feet.*

4. A cask is in the form of the middle zone of a sphere, its top and bottom diameters being 5 feet 8 inches, and its height 5 feet, inside measure; how many gallons will it contain?

Ans. 1193¾ *gallons.*

PROBLEM XI.

To find the convex surface of a sphere, also of a segment and zone thereof.

For the sphere.

RULE.—Multiply the square of the diameter by 3·1416.

For the segment or zone

RULE.—Multiply the circumference of the whole sphere by the height of the segment or zone.

FORMULÆ.

$$A = d^2\pi = cd = \frac{c^2}{\pi} \text{ for the sphere.}$$

$$A = dh\pi = ch \text{ for the segment or zone.}$$

EXAMPLES.

1. Required the convex surface of a sphere, the diameter of which is 2 feet.

$2^2 \times 3{\cdot}1416 = 12{\cdot}4264$ square feet.

2. The circumference of a spherical stone is 4 feet, required its convex surface.

Here $\frac{c^2}{\pi} = 4^2 \div 3{\cdot}1416 = 5{\cdot}0928$ square feet.

3. Required the area of the surface of the earth, its diameter or axis being $7957\frac{3}{4}$ miles, and its circumference 25000 miles.
Ans. 198943750 *square miles.*

4. The axis of a sphere being 42 inches, what is the convex superficies of the segment, whose height is 9 inches?
Ans. 1187·5248 *square inches.*

5. Required the convex surface of a spherical zone, the breadth or height of which is 2 feet, and cut from a sphere of $12\frac{1}{2}$ feet of diameter. *Ans.* 78·54 *square feet.*

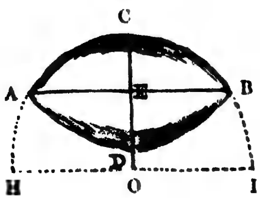
PROBLEM XII.

To find the solidity of a circular spindle.

RULE.—From $\frac{1}{6}$ of the cube of the length A B subtract twice the product of the area of the circular segment A C B and the central distance E O, and multiply the difference by 3·1416 for the solidity.

FORMULA.

$$S = (\tfrac{1}{6}\,AB^3 - 2\,EO \times \text{area of seg. } ACB)\,\pi.$$



NOTE. The central distance E O, and the area of the segment A C B may be found by the Problems for the purpose in Parts II. and III.; however, the whole operation shall be given in the following example.

EXAMPLES.

1. The length A B of a circular spindle is 24 feet, and its middle diameter C D = 18: what is its content?

$$\text{Diameter } HI = \frac{AE^2}{CE} + CE = \frac{12^2}{9} + 9 = 25 \text{ feet.}$$

Then, by Rule II., Prob. VIII., page 45,

$9 \div 25 = \cdot 36 =$ tabular height or versed sine.

The area of corresponding seg. Tab. I., is ·25455, which multiplied by $H I^2 = 25^2$ gives $159 \cdot 09375 =$ area of seg. A C B.

Now $E O = C O - C E = \frac{25}{2} - 9 = 3 \cdot 5$, or $2 E O = 7$.

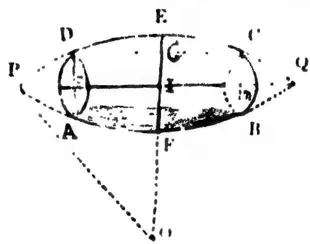
Whence $S = (\frac{1}{6} A B^3 - 2 E O \times$ area of seg. A C B) $\pi =$ $(\frac{1}{6} 24^3 - 7 \times 159 \cdot 09375) \times 3 \cdot 1416 = 3739\frac{1}{2}$ cubic feet.

2. The length of a circular spindle is 6 feet, and its middle diameter $2\frac{1}{2}$ feet; required the content? *Ans.* $16\frac{1}{2}$ *c. ft. nearly.*

PROBLEM XIII.

To find the content of the middle zone or frustrum of a circular spindle.

RULE.—From thrice the square of the length of the whole spindle subtract the square of the length of the middle frustrum, and multiply the differerce by $\frac{1}{4}$ of the said length of the frustrum. Again, multiply four times the central distance by the area of the middle section A D E C B F.—Subtract the latter product from the former, and multiply the remainder by ·7854 for the content.



NOTE.—The middle frustrum of a circular spindle is one of the varieties of casks in guaging, the practical rule for finding the content of which, though not so strictly accurate, is much more concise than the one just given. (See *Guaging*). We shall, therefore, not give an example worked out at length, as in the preceding Problems, but shall give, for the exercise of the student, the following

EXAMPLE.

How many gallons (Imperial) are contained in a cask, in the form of the middle frustrum of a circular spindle, its head diameter (A D) being 12, its bung diameter (E F) 16, and its length (D C) 20 inches? *Ans.* 12·6 *gallons.*

PROBLEM XIV.

To find the convex surface of a circular spindle A C B D, (fig. to Prob. XII.) *or of any segment* A P D, *or zone* A D C B, (fig. to Prob. XIII.).

FORMULÆ.

Let $l =$ length of the spindle, segment, or zone, $a =$ its revolving arc, $r =$ the radius of the generating circle, and $c =$ the central distance: then $A = 2(l r - a c)\pi$.

EXAMPLE.

What is the surface of a circular spindle, the length of which is 24, and its middle breadth 18 feet? *Ans.* $1181\frac{1}{4}$ *sq. ft.*

PROBLEM XV.

To find the solidity of a cylindrical ring.

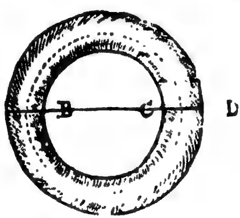
To the thickness of the ring, add the inner diameter; then multiply that sum by the square of the thickness, and the product again by 2·4674, or $\frac{1}{4}$ of the square of 3·1416, for the solidity.

FORMULA.

Let R and r be the inner and outer radii of the ring; then

$$S = \tfrac{1}{4}(R - r)^2 \, (R + r)\pi^2.$$

EXAMPLES.



1. Required the solidity of a ring, the inner diameter being 12 and the thickness 2 inches.

$(12 + 2) \times 2^2 \times 2·4674 =$ 138·1744 square inches.

2. What is the content of a ring, the thickness of which is 4, and inner diameter 16 feet? *Ans.* 789·568 *square feet.*

PROBLEM XVI.

To find the surface of a cylindrical ring.

RULE.—Multiply the sum of the outer and inner radii of the ring by their difference, or the thickness of the ring, and the product by 9·8696, or the square of 3·1416 for the surface.

FORMULA.

$$A = (R + r)\,(R - r)\,\pi^2.$$

EXAMPLES.

1. The inner and outer radii of a ring are 6 and 8 inches: required the area of its surface.

$(8 + 6)\,(8 - 6) \times 9·8696 = 276·3188$ *sq. inches. Ans.*

2. What is the surface of a ring, the inner diameter of which is 16, and thickness 4 inches? *Ans.* 789·568 *sq. in.*

PROBLEM XVII.

To find the solidity of a spheroid.

Square the revolving axis, multiply that square by the fixed axis, and multiply the product by ·5236 for the content.

FORMULA.

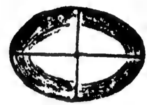
$S = \frac{1}{6} r^2 f \pi$, r being the revolving axis, and f the fixed axis.

NOTE. In the prolate spheroid, the revolving axis is the conjugate; but, in the oblate, the revolving axis is the transverse.

EXAMPLES.

1. Required the solidity of the prolate spheroid A C B D, the axes being A B 50 and C D 30.

30	·5236
30	45000
900	26180000
50	20944
45000	23562·0000 *Ans.*



2. Required the content of an oblate spheroid, the axes of which are 50 and 30 inches. *Ans.* 22·7257 *cubic feet.*

3. What is the solidity of a prolate spheroid, the axes of which are 9 and 7 feet? *Ans.* 230·9076 *cubic feet.*

4. The earth is an oblate spheroid, the equatorial and polar axes being $7925\frac{1}{4}$ and 7899 miles respectively; what is its content in cubic miles?

PROBLEM XVIII.

To find the content of the middle zone of a spheroid, the ends being circular and perpendicular to the fixed axes.

RULE.—To double the square of the middle diameter, add the square of the diameter of one end; multiply this sum by the length of the frustrum, and the product again by ·2618 for the content.

NOTE. The middle zone or frustrum of a spheroid is one of the varieties of casks in guaging.

EXAMPLES.

1. Required the content of the middle zone of a spheroid, the middle diameter being 30, the diameter of each end 18, and the length 40 inches. *Ans.* 22242·528 *cubic inches.*

2. How many gallons are contained in a cask in the form of the middle zone of a spheroid, the bung and head diameters being 20 and 16 inches, and the length 18 inches, all internal measures? *Ans.* $17\frac{13}{14}$ *imperial gallons.*

PROBLEM XIX.

To find the solidity of a paraboloid.

NOTE. A paraboloid is generated by the revolution of a parabola on its axis.

RULE.—Multiply the square of the diameter of the base by the height or axis, and the product again by ·3927 for the content.

EXAMPLE.

What is the content of the parabolic conoid, the height of which is 42, and the diameter of its base 24?

Ans. 9500·1984.

PROBLEM XX.

To find the solidity of the frustrum of a paraboloid.

RULE.—Square the diameter of the two ends, add those two squares together, multiply that sum by the height, and the product again by ·3927, for the content.

EXAMPLES.

1. Required the content of a paraboloidal frustrum, the diameters being 20 and 40, and the height 22½?

Ans. 17671½ *cubic inches.*

2. What is the content of the frustrum of a paraboloid, the greatest diameter being 30, the least 24, and the altitude 9?

Ans. 5216·6268.

PROBLEM XXI.

To find the superficies or solidity of any regular body.

RULE I.—Multiply the proper tabular area (taken from the following table) by the square of the linear edge of the solid, for the superficies.

RULE II.—Multiply the tabular solidity by the cube of the linear edge, for the solid content?

SURFACES AND SOLIDITIES OF REGULAR BODIES.

No. of faces.	Names.	Surfaces.	Solidities.
4	Tetrahedron	1·73205	0·11785
6	Hexahedron or Cube	6·00000	1·00000
8	Octahedron	3·46410	0·47140
12	Dodecahedron	20·64573	7·66312
20	Icosahedron	8·66025	2·18169

NOTE. These are frequently called the Platonic bodies.

EXAMPLES.

1. If the linear edge or side of a tetrahedron be 3 feet, required its surface and solidity?

$1{\cdot}73205 \times 3^2 = 15{\cdot}58845$ *square feet* = surface.
$0{\cdot}11785 \times 3^3 = 3{\cdot}18195$ *cubic feet* = solidity.

2. What are the superficies and solidity of the octahedron, the linear side of which is 2 feet?

Ans. { Superficies 13·8564 *square feet.*
{ Solidity 3·7712 *cubic feet.*

3. What are the surfaces and solidities of a dodecahedron and icosahedron, each of the linear sides of which are 2 feet?

Ans.	Dodecahedron ...	Surface ......82·583 *square feet.*
		Solidity......61·305 *cubic feet.*
	Icosahedron	Surface ......34·641 *square feet.*
		Solidity......17·453 *cubic feet.*

THE SLIDING OR CARPENTER'S RULE.

This instrument is otherwise called the sliding rule, and it is made use of in measuring of timber and artificers' works, both for taking the dimensions, and computing the contents.

The instrument consists of two equal pieces, each a foot in length, which are connected together by a folding joint.

One side or face of the rule, is divided into inches, and half quarters or eighths. On the same face also are several plane scales divided into 12th parts by diagonal lines; which are used in planning dimensions that are taken in feet and inches. The edge of the rule is commonly divided decimally, or into tenths; namely, each foot into 10 equal parts, and each of these into 10 parts again: so that, by means of this latter scale, dimensions are taken in feet, in tenths and hundredths, and then multiplied as common decimal numbers, which is the best way.

On the one part of the other face are four lines, marked A, B, C, D: the two middle ones, B and C, being on a slider, which runs in a groove made in the stock. The same divisions serve for both these two middle lines, the one being above the numbers, and the other below.

These four lines are logarithmic ones, and the three A, B, C, which are all equal to one another, are double lines, as they proceed twice over from 1 to 10. The other or lowest line D, is a single one, proceeding from 4 to 40. It is also called the

4

On the other part of this face there is a table of the value of a load, or 50 cubic feet of timber, at all prices, from 6 pence to 2 shillings a foot.

When 1 at the beginning of any line is accounted 1, or unit, then the 1 in the middle will be 10, and the 1 at the end 100; and when 1 at the beginning is accounted 10, then the 1 in the middle is 100, and the 1 at the end 1000; and so on. All the smaller divisions being altered proportionally.

Problem I.

To multiply numbers together.

Suppose the two numbers 13 and 24.—Set 1 on B to 13 on A; then against 24 on B stands 312 on A, which is the required product of the two given numbers 13 and 24.

Note. In any operations, when a number runs beyond the end of the line, seek it on the other radius, or other part of the line; that is, take the 10th part of it, or the 100th part of it, &c., and increase the result proportionally 10 fold, or 100 fold, &c.

In like manner the product of 35 and 19 is 665.
And the product of 270 and 54 is 14580.

Problem II.

To divide by the sliding rule.

As suppose to divide 312 by 24.—Set the divisor 24 on B to the dividend 312 on A; then against 1 on B stands 13, the quotient, on A.

Also 396 divided by 27 gives 14·6.
And 741 divided by 42 gives 17·6.

Problem III.

To square any number.

Suppose to square 23.—Set 1 on B to 23 on A; then against 23 on B, stands 529 on A, which is the square of 23.

Or, by the other two lines, set 1 or 100 on C to the 10 on D, then against every number on D, stands its square in the line C. So, against 23 stands 529
against 20 stands 400
against 30 stands 900
and so on.

If the given number be hundreds, &c., reckon the 1 on D for 100, or 1000, &c., then the corresponding 1 on C is 10000, or 100000, &c. So the square of 230 is found to be 52900.

Problem IV.

To extract the square root.

Set 1 or 100, &c., on C to 1 or 10, &c., on D; then against every number found on C, stands its square root on D.

Thus, against 529 stands its root 23
against 400 stands its root 20
against 900 stands its root 30
against 300 stands its root 17.3
and so on.

Problem V.

To find a mean proportional between two numbers.

As suppose between 29 and 430.—Set the one number 29 on C to the same on D: then against the other number 430 on C, stands their mean proportional 111 on D.

Also the mean between 29 and 320 is 96·3.

And the mean between 71 and 274 is 139.

Problem. VI.

To find a third proportional to two numbers.

Suppose to 21 and 32.—Set the first 21 on B to the second 32 on A: then against the second 32 on B stands 48·8 on A; which is the third proportional sought.

Also the third proportional to 17 and 29 is 49·4.

And the third proportional to 73 and 14 is 2·5.

Problem VII.

To find a fourth proportional to three numbers: or, to perform the Rule-of-Three.

Suppose to find a fourth proportional to 12, 28, and 114.—Set the first term 12 on B to the second term 28 on A; then against the third term 114 on B, stands 266 on A, which is the fourth proportional sought.

Also the fourth proportional to 6, 14, 29, is 67·6.

And the fourth proportional to 27, 20, 73, is 54·0.

TIMBER MEASURING.

Problem I.

To find the area, or superficial content, of a board or plank.

Multiply the length by the mean breadth.

Note. When the board is tapering, add the breadth at the two ends together, and take half the sum for the mean breadth.

BY THE SLIDING RULE.

Set 12 on B to the breadth in inches on A; then against the length in feet on B, is the content on A, in feet and fractional parts.

EXAMPLES.

1. What is the value of a plank, at $1\frac{1}{2}d.$ per foot, whose length is 12 feet 6 inches, and mean breadth 11 inches?

By decimals.			By duodecimals.		
	12·5			12	6
	11				11
12	137·5	$1\frac{1}{2}d.$ is $\frac{1}{8}$	11	5 . 6	
	11·46				
$1\frac{1}{2}d.$ is $\frac{1}{8}$	1s. 5d. *Ans.*		1s.	$4\frac{1}{2}d.$	
		5 *in.* is $\frac{5}{12}$	0	$\frac{1}{2}$	
			1s.	5d. *Ans.*	

BY THE SLIDING RULE.

As 12 B : 11 A : : $12\frac{1}{2}$ B : $11\frac{1}{2}$ A.

That is, as 12 on B is to 11 on A, so is $12\frac{1}{2}$ on B to $11\frac{1}{2}$ on A.

EXAMPLES.

2. Required the content of a board, whose length is 11 feet 2 inches, and breadth 1 foot 10 inches.

Ans. 20 *feet* 5 *inches* 8''.

3. What is the value of a plank, which is 12 feet 9 inches long, and 1 foot 3 inches broad, at $2\frac{1}{2}d.$ a foot?

Ans. 3s. $3\frac{3}{4}d.$

4. Required the value of five oaken planks at 3*d.* per foot, each of them being $17\frac{1}{2}$ feet long; and their several breadths are as follow, namely, two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{2}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ at the narrower.

Ans. £1 5s. $8\frac{1}{4}d.$

PROBLEM II.

To find the solid content of squared or four-sided timber.

Multiply the mean breadth by the mean thickness, and the product again by the length, and the last product will give the content.

BY THE SLIDING RULE.

C D D C

As length : 12 or 10 : : quarter girt : content.

That is as the length in feet on C, is to 12 on D when the quarter girt is in inches, or to 10 on D when it is in tenths of feet; so is the quarter girt on D, to the content on C.

NOTE 1. If the tree taper regularly from the one end to the other, either take the mean breadth and thickness in the middle, or take the dimensions at the two ends, and half their sum for the mean dimension.

NOTE 2. If the piece do not taper regularly, but is unequally thick in some parts and small in others; take several different dimensions, add them all together; and divide their sum by the number of them, for the mean dimension.

NOTE 3. The quarter girt is a geometrical mean proportional between the mean breadth and thickness, that is the square root of their product. Sometimes unskilful measurers use the arithmetical mean instead of it, that is half their sum; but this is always attended with error, and the more so, as the breadth and depth differ the more from each other.

EXAMPLES.

1. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and less end 1 foot 6 inches and 1 foot 3 inches, and the thickness at the greater and less end 1 foot 3 inches and 1 foot: required the solid content.

Decimals.		Duodecimals.			
1·5		1	6		
1·25		1	3		
2) 2·75		2) 2	9		
1·375	mean breadth	1	4	6	
1·25		1	3		
1· 0		1	0		
2) 2·25		2) 2	3		
1·125	mean depth	1	1	6	
1·375	mean breadth	1	4	6	
5625		1	1	6	
7875			4	6	
3375				6	9
1125					
1·5468875		1	6	6	9
18·5	length	18	6		
7734375					
12375000		27	10	1	6
1546875			9	3	4
28·6171875	content	28	7	4	10

BY THE SLIDING RULE.

	B		A		B		A	
As	1	:	$13\frac{1}{2}$	::	$16\frac{1}{2}$	:	22·3,	the mean square.
	C		D		C		D	
As	1	:	1	::	223	:	14·9,	quarter girt.
	C		D		D		C	
As	$18\frac{1}{2}$	:	12	::	14·9	:	28·6,	the content.

2. What is the content of the piece of timber, whose length is 24½ feet, and the mean breadth and thickness each 1·04 feet?

Ans. 26½ *feet.*

3. Required the content of a piece of timber, whose length is 20·38 feet, and its ends unequal squares, the side of the greater being 19⅛, and the side of the less 9⅞ inches.

Ans. 29·756 *feet.*

4. Required the content of the piece of timber, whose length is 27·36 feet; at the greater end the breadth is 1·78, and the thickness 1·23; and at the less end the breadth is 1·04, and thickness 0·91.

Ans. 41·278 *feet.*

Problem III.

To find the solidity of round or unsquared timber.

Rule I., or common rule.—Multiply the square of the quarter girt, or of ¼ of the mean circumference, by the length, for the content.

BY THE SLIDING RULE.

As the length upon C : 12 or 10 upon D : : quarter girt, in 12ths or 10ths, on D : content on C.

Note 1. When the tree is tapering, take the mean dimensions as in the former Problems, either by girting it in the middle, for the mean girt, or at the two ends, and take half the sum of the two. But when the tree is very irregular, divide it into several lengths, and find the content of each part separately: or else, add all the girts together, and divide the sum by the number of them, for the mean girt.

Note 2. This rule, which is commonly used, gives the answer about ¼ less than the true quantity in the tree, or nearly what the quantity would be after the tree is hewed square in the usual way; so that it seems intended to make an allowance for squaring the tree. When the true quantity is desired, use the 2nd Rule given below.

EXAMPLES.

1. A piece of round timber being 9 feet 6 inches long, and its mean quarter girt 42 inches; what is the content?

Decimals.		Duodecimals.		
3·5	quarter girt	3	6	
3·5		3	6	
175		10	6	
105		1	9	
12·25		12	3	
9·5	length	9	6	
6125		110	3	
11025		6	1	6
116·375	content	116	4	6

BY THE SLIDING RULE.

	C		D		D		C
As	9·5	:	10	: :	35	:	116¼
Or	9·5	:	12	: :	42	:	116¼

2. The length of a tree is 24 feet, its girt at the thicker end 14 feet, and at the smaller end 2 feet; required the content. *Ans.* 96 *feet.*

3. What is the content of a tree, whose mean girt is 3·15 feet, and length 14 feet 6 inches? *Ans.* 8·9929 *feet.*

4. Required the content of a tree, whose length is 17¼ feet, which girts in five different places as follows, namely, in the first place 9·43 feet, in the second 7·92, in the third 6·15, in the fourth 4·74, and in the fifth 3·16. *Ans.* 42·5195.

Rule II.—Multiply the square of ⅕ of the mean girt by double the length, and the product will be the content, very near the truth.

BY THE SLIDING RULE.

As the double length on C : 12 or 10 on D : : ⅕ of the girt, in 12ths or 10ths, on D : content on C.

EXAMPLES.

1. What is the content of a tree, its length being 9 feet 6 inches, and its mean girt 14 feet?

Decimals.		Duodecimals.		
2·8	⅕ of girt	2	9	7
2·8		2	9	7
224		5	7	2
56		2	1	3
			1	8
7·84				
19		7	10	1
7056		19		
784				
148·96	content	148	11	7

BY THE SLIDING RULE.

	C		D		D		C
As	19	:	10	: :	28	:	149
Or	19	:	12	: :	$33\frac{6}{10}$	:	149

2. Required the content of a tree, which is 24 feet long, and mean girt 8 feet. *Ans.* 122·88 *feet.*

3. The length of a tree is $14\frac{1}{2}$ feet, and mean girt 3·15 feet; what is the content? *Ans.* 11·51 *feet.*

4. The length of a tree is $17\frac{1}{4}$ feet, and its mean girt 6·28; what is the content? *Ans.* 54·425 *feet.*

ARTIFICERS' WORK.

Artificers compute the contents of their works by several different measures.

As glazing by the square foot.

Painting, masonry, plastering, paving, &c., by the yard of 9 square feet.

Flooring, partitioning, roofing, tiling, &c., by the square of 100 square feet.

And brick-work, either by the yard of 9 square feet, or by the square rod of $272\frac{1}{4}$ square feet, or $30\frac{1}{4}$ square yards, being the square of the rod of $16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards long.

As this number $272\frac{1}{4}$ is a troublesome number to divide by, the $\frac{1}{4}$ is often omitted in practice, and the content in feet divided only by the 272. But when the exact divisor $272\frac{1}{4}$ is to be used, it will be easier to multiply the feet by 4, and then divide successively by 9, 11, and 11. Also to divide square yards by $30\frac{1}{4}$, first multiply them by 4, and then divide twice by 11.

All works, whether superficial or solid, are computed by the rules proper to the figure of them, whether it be a triangle, or rectangle, a parallelopiped, or any other figure.

BRICKLAYERS' WORK.

Brick-work is usually estimated by the rod at the rate of a brick and a half thick; so that if a wall be more or less than this standard thickness, it must be reduced to it, as follows: Multiply the superficial content of the wall by the number of half bricks in thickness, and divide the product by 3. And to find the superficial content of a wall, multiply the length by the height in feet, and divide the product by 272, for the content in rods.

Chimneys are by some measured as if they were solid, deducting only the vacuity from the hearth to the mantle, on account of the trouble of them.

All windows, doors, &c., are to be deducted out of the contents of the wall in which they are placed, and a distinct charge must be made for window heads, sills, quoins, &c.

A rod of brickwork of standard thickness contains about 305 cubic feet, and 4500 bricks, making due allowance for mortar, joints, &c. Assuming the cost of bricks at the kiln to be 36*s.* per thousand, the cost of bricks per rod will be 36*s.* × 4½ = £8 2*s.* 0*d.* The cost for building a rod of brickwork may be taken at 36*s.* The cost of cartage and mortar varies with the locality of the building, this may be taken to average about 20*s.* per rod. These items being collected, there results—

	£	*s.*	*d.*
Cost of bricks.............. =	8	2	0
Cost of labour .... =	1	16	0
Cost of cartage and mortar =	1	0	0
	10	18	0
Profit at 10 per cent....... =	1	1	10
Cost per rod.............. =	11	19	10, or £12 per rd.

NOTE. The student must, of course, make his estimation according to the locality of the building, this being only a specimen of the method adopted by intelligent architects and builders.

EXAMPLES.

1. How many rods of standard brickwork are in a wall whose length or compass is 57 feet 3 inches, and heigth 24 feet 6 inches; the walls being 2½ bricks, or 5 half bricks thick?

Decimals.		Duodecimals.		
57·25		57	3	
24·5		24	6	
28625		234	0	
22900		114		
11450		28	7	6
1402·625		1402	7	6
5	half bricks			5
3 | 7013·125		3 | 7013	1	6
272 | 2337·708⅓	square feet.	272 | 2337	8	6
8·594 *rods.*		*8r.* 161*ft.* 8	6″	

2. A triangular gable is raised 17½ feet high, on an end wall whose length is 24 feet 9 inches, the thickness being 2 bricks; required the reduced content in square yards. *Ans.* 32·08⅓ *yds.*

3. The end wall of a house is 28 feet 10 inches long, and 55 feet 8 inches high to the eaves, 20 feet high is $2\frac{1}{2}$ bricks thick, other 20 feet high is 2 bricks thick, and the remaining 15 feet 8 inches is $1\frac{1}{2}$ brick thick, above which is a triangular gable of 1 brick thick, which rises 42 courses of bricks, of which every four courses make a foot. What is the whole content, and cost at £12 per rod? *Ans.* { 235·62 *yards*, and 8·384 *rods*. Cost £100 12*s*. 2*d*.

MASONS' WORK.

To masonry belongs all sorts of stone-work; and the measure made use of is a square foot or square yard.

Walls, columns, blocks of stone or marble, &c., are measured by the cubic foot, and pavements, slabs, chimney-pieces, &c., by the superficial or square foot.

Cubic or solid measure is used for the materials, and square measure for the workmanship.

In the solid measure, the true length, breadth, and thickness, are taken, and multiplied continually together. In the superficial, there must be taken the length and breadth of every part of the projection, which is seen without the general upright face of the building.

The cost of stones for walling varies with the locality, as already noticed in bricklayers' work.

A square yard of rubble walling 2 feet thick weighs $1\frac{1}{4}$ tons, and assuming the cost of rubble stones at the quarry to be 6*d*. per ton, the cost of cartage 1*s*. per ton, the cost of mortar 4*d*., and that of labour 1*s*. 8*d*. per square yard, we shall have the following

Estimate for a square yard of rubble walling 2 feet in thickness.

		s.	*d.*
Cost of materials	$= 6d. \times 1\frac{1}{4} =$	0	$7\frac{1}{2}$
Cost of cartage	$= 1s. \times 1\frac{1}{4} =$	1	3
Cost of labour	$=$	1	8
Cost of mortar	$=$	0	4
		3	$10\frac{1}{2}$
Profit at 10 per cent....................	$=$	0	$4\frac{1}{2}$
Total cost per square yard	$=$	4	3

An additional charge must be made for quoins, window heads and sills, when used in houses constructed of rubble work: also,

when the fronts of houses are constructed entirely with Ashler work, a separate estimate must be made for it. Ashlers usually average 9 inches in the bed, this width must, therefore, be deducted from whole width of the wall, and the remainder estimated as rubble work, to which the additional cost at the quarry, and of hewing the ashler must be added.

EXAMPLES.

1. Required the square yards and cost of a rubble wall of the specified thickness, the length of which is 53 feet 6 inches, and the height 12 feet 3 inches, at 4*s.* 3*d.* per square yard.

$$53 \cdot 5 \times 12 \cdot 25 \div 9 = 72 \cdot 82 \text{ square yards},$$

$$\text{and } 72 \cdot 82 \times 4s.\ 3d. = £15\ 9s.\ 5\tfrac{3}{4}d. \text{ cost.}$$

BY THE SLIDING RULE.

B A B A

$9 : 53\frac{1}{2} :: 12\frac{1}{4} : 72\frac{3}{4}$ square yards.

2. Required the value of a marble slab, at 8*s.* per foot; the length being 5 feet 7 inches, and breadth 1 foot 10 inches.

Ans. £4 1*s.* 10$\frac{1}{4}$*d.*

3. In a chimney piece, suppose the length of the mantle and

slab, each	4ft.	6in.
breadth of both together	3	2
length of each jamb	4	4
breadth of both together	1	9

required the superficial content. *Ans.* 21*ft.* 10*in.*

CARPENTERS' AND JOINERS' WORK.

To this branch belongs all the wood-work of a house, such as flooring, partitioning, roofing, &c.

The large and plain parts are usually measured by the square of 100 feet; but enriched mouldings, and some other articles, are often estimated by running or lineal measure, and some things are rated by the piece.

Joists are measured by, multiply the depth, breadth, and length all together, for the content of one joist; multiply that by the number of the joists. Note, that the length of the joists will exceed the breadth of the room by the thickness of the wall and $\frac{1}{4}$ of the same, because each end is let into the wall about $\frac{3}{4}$ of its thickness.

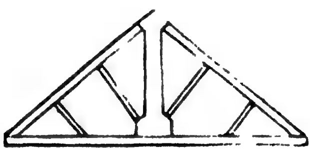
Partitions are measured from wall to wall for one dimension,

and from floor to floor, as far as they extend, for the other; then multiply the length by the height.

In measuring joiners' work, the string is made to ply close to every part of the work over which it passes.

In roofing, the length of the rafters is equal to the length of a string stretched from the ridge down the rafter, and along the eaves-board, till it meets with the top of the wall. This length multiplied by the common depth and breadth of the rafters, gives the content of one, and that multiplied by the number of them, gives the content of all the rafters.

King-post roofs, &c., all the timbers in a roof are measured in the same manner as the joists, &c., in flooring. In the annexed figure, representing a truss for a roof, all the beams, as the tie-beam, king-post, braces, &c., are measured to their full lengths, breadths, and thicknesses, including the lengths of tenons; also the parts cut out on each side of the king-post, to form abutments for the braces, are included; unless their lengths exceed 2 feet each by 3 inches breadth, when their solidities must be deducted, pieces of smaller size, being considered of little or no value, are, therefore, included in the measurement.



For stair-cases, take the breadth of all the steps, by making a line ply close over them, from the top to the bottom; and multiply the length of this line by the length of a step for the whole area.—By the length of a step, is meant the length of the front and the returns at the two ends; and by the breadth, is to be understood the girt of its two outer surfaces, or the tread and rise.

For the balustrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel post, for one dimension; and twice the length of the baluster upon the landing, with the girt of the hand-rail, for the other dimension.

For wainscoting, take compass of the room for one dimension; and the height from the floor to the ceiling, making the string ply close into all the mouldings, for the other dimension. —Out of this must be made deductions, for windows, doors, and chimneys, &c.

For doors, it is usual to allow for their thickness, by adding it into both the dimensions of length, and breadth, and then

multiply them together for the area.—If the door be pannelled on both sides, take double its measure for the workmanship: but if one side only be pannelled, take the area and its half for the workmanship.—*For the surrounding architrave*, girt it about the outermost part for one dimension, and measure over it as far as it can be seen when the door is open, for the other.

Window-shutters, *bases*, *&c.*, are measured in the same manner.

EXAMPLES.

1. Required the content of a floor 48 feet 6 inches long, and 24 feet 3 inches broad.

Decimals.	Duodecimals.
48·5	48 6
24¼	24 3
1940	204 0
970	96
12·125	12 1 6
1176·125 feet	1176 1 6
11·76125 squares	*Ans.* 11·76 1 6

2. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad, how many squares are in it? *Ans. 5 squares* 98⅛ *feet.*

3. How many squares are there in 173 feet 10 inches in length, and 10 feet 7 inches height, of partitioning?

Ans. 18·3972 *squares.*

4. What cost the roofing of a house at 10*s.* 6*d.* a square: the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches: reckoning the roof $\frac{3}{2}$ of the flat?

Ans. £12 12*s.* 11¾*d.*

5. To how much, at 6*s.* per square yard, amounts the wainscoting, of a room; the height, taking in the cornice and mouldings being 12 feet 6 inches, and the whole compass 83 feet 8 inches; also the three window shutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches: the door and shutters, being worked on both sides, are reckoned work and half work? *Ans.* £36 12*s.* 2¼*d.*

6. In a naked floor there are 2 girders, each 20 feet long, and 1 foot 2 inches by 1 foot: there are 16 bridging joists, each 20 feet long, and 6½ inches by 3; 16 binding joists, each 9 feet in length, and 8½ inches by 4: 48 ceiling joists, each 6 feet long, and 4 inches by 2½: required the content in cubic feet.

Ans. 144 *cubic feet.*

7. **What will the wainscoting of a room cost at 4*s.* per square yard; the height of the room, including cornice and the mouldings, is 12½ feet and the compass 125½ feet; there are three window shutters, each 7 feet 8 inches by 3½ feet, and the door 7 feet by 3½ feet; the door and shutters, being worked on both sides, are reckoned half work additional?** *Ans.* £36 12*s.* 2½.

SLATERS' AND TILERS' WORK.

In these articles, the content of a roof is found by multiplying the length of the ridge by the girt over from eaves to eaves; making allowance in this girt for the double row of slates at the bottom, or for how much one row of slates or tiles is laid over another.

When the roof is of a true pitch, that is, forming a right angle at top; then the breadth of the building with its half added, is the girt over both sides.

In angles formed in a roof, running from the ridge to the eaves, when the angle bends inwards, it is called a valley; but when outwards, it is called a hip.

Deductions are made for chimney shafts or window holes.

1. Required the content of a slated roof, the length being 45 feet 9 inches, and the whole girt 34 feet 3 inches.

Decimals.	Duodecimals.
45·75	45 9
34¼	34 3
18300	205 6
13725	135
114375	11 5 3
9)1566·9375 feet	9)1566 11 3
yards 174·104	174 *yds.* 11 3′.

2. **To how much amounts the tiling of a house, at 25*s.* 6*d.* per square; the length being 43 feet 10 inches, and the breadth on the flat 27 feet 5 inches, also the eaves projecting 16 inches on each side, and the roof of a true pitch?** *Ans.* £24 9*s.* 5¼*d.*

PLASTERERS' WORK.

Plasterers' work is of two kinds, namely, ceiling, which is plastering upon laths; and rendering, which is plastering upon walls: which are measured separately.

The contents are estimated either by the foot or yard, or square of 100 feet. Enriched mouldings, &c., are rated by running or lineal measure.

Deductions are to be made for chimneys, doors, windows, &c.

EXAMPLES.

1. How many yards contains the ceiling, which is 43 feet 3 inches long, and 25 feet 6 inches broad?

Decimals.	Duodecimals.
43·25	43 3
25½	25 6
21625	221 3
8650	86
21625	21 7 6
9) 1102·875	9) 1102 10 6
yards 122·541	*Ans.* 122 *yds.* 4 *ft.* 10′ 6″.

2. To how much amounts the ceiling of a room, at 10*d.* per yard; the length bei.g 21 feet 8 inches, and the breadth 14 feet 10 inches? *Ans.* £1 9*s.* 8¾*d.*

3. The length of a room is 18 feet 6 inches, the breadth 12 feet 3 inches, and height 10 feet 6 inches; to how much amounts the ceiling and rendering, the former at 8*d.* and the latter at 3*d.* per yard; allowing for the door of 7 feet by 3 feet 8, and a fire-place of 5 feet square? *Ans.* £1 13*s.* 3*d.*

4. Required the quantity of plastering in a room the length being 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, which girts 8½ inches, and projects 5 inches from the wall on the upper part next the ceiling; deducting only for a door 7 feet by 4.

Ans. { 53 *yds.* 5 *ft.* 3 *in.* *of rendering.*
18 5 6 *of ceiling.*
39 0¼ *of cornice.*

PAINTERS' WORK.

Painters' work is computed in square yards. Every part is measured where the colour lies; and the measuring line is forced into all the mouldings and corners.

Windows are done at so much a piece; and it is usual to allow double measure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting contains the room which is 65 feet 6 inches in compass, and 12 feet 4 inches high?

Ans. 89 *yards* 6 *feet* 10′.

2. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows each 6 feet by 3 feet 2 inches? *Ans.* $73\frac{2}{27}$ *yds.*

3. What cost the painting of a room at 6*d.* per yard; its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; also the door is 7 feet by 3 feet 6, and the window shutters to two windows each 7 feet 9 by 3 feet 6, but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep: deducting the fire-place of 5 feet by 5 feet 6? *Ans.* £3 3*s.* $10\frac{1}{2}$*d.*

GLAZIERS' WORK.

Glaziers take their dimensions either in feet, inches, and parts, or feet, tenths and hundredths. And they compute their work in square feet.

In taking the length and breadth of a window, the cross bars between the squares are included. Also windows of round or oval forms are measured as square, measuring them to their greatest length and breadth, on account of the waste in cutting the glass.

EXAMPLES.

1. How many square feet contains the window which is 4·25 feet long, and 2·75 feet broad?

Decimals.		Duodecimals.		
2·75		2	9	
$4\frac{1}{4}$		4	3	
11·00		11	0	
·6875			8	3
11·6875	*Ans.*	11	8	3

2. What will the glazing a triangular sky-light come to at 10*d.* per foot; the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches? *Ans.* £1 15*s.* 1¾*d.*

3. There is a house with three tier of windows, three windows in each tier, their common breadth 3 feet 11 inches;

now the height of the first tier is	7 feet	10 inches,
of the second	6	8
of the third	5	4

Required the expense of glazing at 14*d.* per foot.

Ans. £13 11*s.* 10½*d.*

4. Required the expense of glazing the windows of a house at 13*d.* a foot; there being three stories, and three windows in each story: the height of the lower tier is 7 feet 9 inches,

of the middle	6	6
of the upper	5	3¼
and of an oval window over the door	1	10½

The common breadth of all the windows being 3 feet 9 inches. *Ans.* £12 5*s.* 6*d.*

PAVIERS' WORK.

Paviers' work is done by the square yard, and the content is found by multiplying the length by the breadth.

EXAMPLES.

1. What cost the paving a foot-path at 3*s.* 4*d.* per yard; the length being 35 feet 4 inches, and breadth 3 feet 3 inches?

Ans. Content 32 *yards* 3 *feet* 6′. Cost £5 7*s.* 11¼*d.*

2. What will be the expense of paving a rectangular court yard, whose length is 63 feet, and breadth 45 feet; in which there is laid a foot path of 5 feet 3 inches broad, running the whole length, with broad stones, at 3*s.* a yard; the rest being paved with pebbles at 2*s.* 6*d.* a yard? *Ans.* £40 5*s.* 10½*d.*

PLUMBERS' WORK.

Plumbers' work is rated at so much a pound, or else by the hundred weight of 112 pounds.

Sheet lead used in roofing, guttering, &c., is from 7 to 12 lbs. to the square foot. And a pipe of an inch bore is commonly 13 or 14 lbs. to the yard in length.

EXAMPLES.

1. How much weighs the lead which is 39 feet 6 inches long, and 3 feet 3 inches broad, at $8\frac{1}{2}$ lbs to the square foot?

Decimals.	Duodecimals.		
39·5	39	6	
$3\frac{1}{4}$	3	3	
118.5	118	6	
9·875	9	10	6
128·375	128	4	6
$8\frac{1}{2}$	$8\frac{1}{2}$		
	1024		
1027·000	64		
64·1875	$2\frac{5}{6}$		
	$0\frac{17}{48}$		
1091·1875 *Ans.*	$1091\frac{9}{48}$ *lbs.*		

2. What cost the covering and guttering a roof with lead, at 18*s.* the cwt.; the length of the roof being 43 feet, and breadth or girt over it 32 feet; the guttering 57 feet long, and 2 feet wide: the former 9·831 lbs., and the latter 7·373 lbs. to the square foot? *Ans.* £115 9*s.* $1\frac{1}{2}$*d.*

ARCHED AND VAULTED ROOFS.

To find the concave surface of a circular, gothic, or elliptical vaulted roof.

RULE.—Multiply the length of the arch by the length of the vault for the required surface.

NOTE. The length of the arch, for most practical purposes, is found by applying a small chord along its concavity, and then measuring its length.

EXAMPLE.

What is the concave surface of Caerfylly bridge, over the Taafe; which is a segment of a circle, span 140, height 35, and width 12 feet? *Ans.* 1944·4 *square feet.*

To find the content of a circular, gothic, or elliptical roof.

RULE.—Multiply the area of the end by the length of the roof for the content of the vacuity.

To find the solid content of the materials.

From the solid content of the whole arch take that of the vacuity for the solid content of the materials.

EXAMPLE.

Required the solidity of the vacuity and of the materials of a

circular vault; span 36 feet, height 18 feet, thickness of walls at the spring 6 feet, thickness of crown 4 feet, and length of the vault 100 feet?

Ans. { 1884·96 *cubic yards solidity of vacuity.*
2026·15 *cubic yards solidity of materials.*

To find the surface and solidity of a dome, the height and dimensions of the base being given.

RULE.—Take twice the area for the surface, and multiply the area of the base by $\frac{2}{3}$ of the height.

NOTE. Although these rules are only true when the domes are hemispherical, yet they are sufficiently near the truth for all practical purposes.

EXAMPLE.

Required the surface and solidity of a hemispherical dome, the diameter of its base being 60 feet.

Ans. { Surface 314·16 *square yards.*
Solidity 2094·4 *cubic yards.*

NOTE 1. The surface of a saloon is found in the same manner as a vaulted roof.

NOTE 2. Rules might have been here given for the measurement of haystacks, coal-heaps, &c.; but these may be readily resolved into two or more of those solids, the methods of finding the content of which are given in the Mensuration of Solids, Part IV. Moreover, haystacks are usually sold by weight, and seldom or never by measurement.

SPECIFIC GRAVITY.

The specific gravity of bodies are their weights when compared with an equal bulk of pure water, which, at the temperature of 40°, weighs 1000 ounces avoirdupois per cubic foot. The following table, therefore, contains the weights of a cubic foot of several bodies in ounces.

A TABLE OF THE SPECIFIC GRAVITY OF BODIES.

Platinum	21470	Light earth	1984
Gold	19260	Solid Gunpowder	1745
Mercury	13600	Sand	1520
Lead	11325	Coal from 1030 to	1300
Silver	10470	Pitch	1150
Copper	9000	Box-wood	1030
Cast brass	8400	Sea-water	1030
Steel	7850	Common water	1000
Iron	7704	Mahogany	1065
Cast Iron	7065	Oak	925
Tin	7320	Ash	755
Granite	9950	Beech	700
Flint Glass	3000	Elm	600
Marble	2700	Fir	540
Freestone	2520	Cork	240
Clay	2160	Air	$1\frac{1}{4}$
Brick	2000		

To find the weight of a body from its bulk.

RULE.—Multiply the content of the body, in cubic feet, by its tabular specific gravity for its weight in avoirdupois ounces.

EXAMPLES.

1. Required the weight of a block of marble, whose length is 63 feet, and breadth and thickness each 12 feet, being the dimensions of one of the stones in the walls of Balbeck.

$$\frac{63 \times 12^2 \times 2700}{16 \times 112 \times 20} = 683\tfrac{7}{16} \text{ tons, which is equal to the}$$

burden of a second rate East India ship.

2. What is the weight of a block of dry oak, which measures 10 feet long, and 3 feet by $2\frac{1}{2}$? *Ans.* 4336 *lb. nearly.*

To find the magnitude of a body from its weight.

RULE.—Divide its weight in avoirdupois ounces by it tabular specific gravity for its content in cubic feet.

EXAMPLES.

1. Required the content of an irregular block of freestone, which weighs 1 cwt.

$$\frac{112 \times 16}{2520} = \text{cubic feet,}$$

$$\text{and } \frac{112 \times 16 \times 1728}{2520} = 1228{\cdot}8 \text{ cubic inches.}$$

2. How many cubic feet are there in a ton of dry oak?
Ans. $38\frac{138}{185}$ *cubic feet.*

3. A cast iron pipe is 6 inches diameter in the bore and 1 inch in thickness; required the weight of a running foot.
Ans. 67·45 *lbs.*

LAND SURVEYING.

DESCRIPTION OF INSTRUMENTS USED FOR MEASURING AND PLANNING SURVEYS.

THE CHAIN.

THE chain, usually called Gunter's chain, is almost generally used in the British dominions, for measuring the distances required in a survey. It is 66 feet, or 4 poles, in length, and is divided into 100 links, which are joined by rings. The length of each link, together with half the rings connecting it with the

adjoining links, is consequently $\frac{66}{100}$ of a foot, or $\frac{66 \times 12}{100} = 7{\cdot}92$ inches. At every tenth link from each end is attached a piece of brass with notches; that at the tenth link has one notch, that at the 20th two notches, that at the 30th three, that at the 40th four, the middle of the chain, or the 50th link being marked with a large round piece of brass; hence, any distance on the chain may be readily counted. Part of the first link, at each end, is formed into a large ring for the purpose of holding it with the hand.

The chain acquires extension by much use, it should, therefore, be frequently examined, and adjusted to the proper length by taking out some of the rings between the links: for this purpose, chains having three rings between each link are to be preferred to those having only two.

THE OFFSET STAFF.

The offset staff is used to measure short distances, called offsets; hence its name. It is usually ten links in length, the links being numbered thereon with the figs. 1, 2, 3, &c. It is usually pointed with iron at one end, for the purpose of fixing it in the ground, as an object for ranging lines, for marking stations, &c.

THE CROSS.

The cross is an instrument used by surveyors to erect perpendiculars. It is usually a round piece of sycamore, box, or mahogany, about four inches in diameter, with two folding sights at right angles to each other, or more commonly with two fine grooves sawed at right angles to each other, which answer the purpose of sights. It is sometimes fixed on a staff of convenient length for use, pointed with iron at the bottom, that it may be fixed firmly in the ground: but it is found more commodious in practice to have a small pocket cross, which may be readily fitted to the offset-staff, either by an iron spike on the cross being inserted in a hole made in the offset-staff, or the offset-staff being passed through a hole made in the cross, to about the eighth link from the piked end, at which place the staff must be shouldered, that the cross may rest firmly.

DIRECTIONS FOR MEASURING LINES ON THE GROUND.

Besides the instruments already described, ten arrows must be provided, about 12 inches long, pointed at the end, so as to be readily pressed into the ground, and turned at the other end, so as to form a ring to serve for a handle.

In using the chain, marks are to be set up at the extremities of the line to be measured, as well as at its intermediate points, if its extremities cannot be seen from one another, on account of hills, woods, hedges, or other obstructions. Two persons are then required by the surveyor to perform the measurement. The chain leader starts with the ten arrows in his left hand, and one end of the chain in his right; while the follower remains at the starting point, who, looking at the staff or staves, that mark the line to be measured, directs the leader to extend the chain in the direction of the staff or staves. The leader then puts down one of his arrows, and proceeds a second chain's length in the same direction, while the follower comes up to the arrow first put down. A second arrow being now put down by the leader, the first is taken up by the follower; and the same operation is repeated till the leader has expended all his arrows. Ten chains, or 1000 links, having now been measured and noted in the field book, the follower returns the ten arrows to the leader, and the same operation is repeated as often as necessary. When the leader arrives at the end of the line, the number of arrows in the follower's hand shews the number of chains measured since the last exchange of arrows noted in the field book, and the number of links extending from the last arrow to the mark or staff at the extremity of the line, being also added, gives the entire measurement of the line. Thus, if the arrows have been exchanged seven times, and if the follower have six arrows, and from the arrow last put down to the end of the line be 83 links, the whole measurement will be 7683 links, or 76 chains 83 links, which is usually written thus—76·83 chains, the two last figures being decimals of a chain.

$$\begin{array}{r} 7000 \\ 600 \\ 83 \\ \hline 7683 \end{array}$$

In using the chain, care must be taken to stretch it always with the same tension, as it will extend by much use, and will therefore require to be examined occasionally, and shortened, if necessary. But a good chain may be used several days on tolerably smooth ground, without any material extension.

The surveyor must mark, or caused to be marked, every station on the line, while it is being measured, with a staff or cross on the ground, entering its distance in the field book.

When a survey is made for a finished plan, all remarkable objects should be noted down; as buildings, roads, rivers, ponds, footpaths, gates, &c.

The boundary of the estate measured ought to be carefully observed. If the ditch be outside the boundary fence, it usually

belongs to the estate, and *vice versa;* although this is not uniformly the case; therefore, inquiry ought to be made with respect to the real boundary.

In some places five links from the hedge-posts or roots of the quickwood are allowed for the breadth of the ditch, but this breadth varies to as far as even ten links, especially in swampy countries.

All ditches and fences must be measured with the fields to which they belong, when the full quantity on the plan is required: but when the growing crops only are to be measured, only so much as is occupied by the crops.

Instruments for Laying Down or Plotting Surveys.

THE COMMON DRAWING COMPASSES.

This instrument consists of two legs moveable about a joint so that the points at the extremities of the legs may be set at any required distance from one another: it is used to transfer and measure distances, and to describe arcs and circles.

Note. There are several other kinds of compasses, used for planning; as those with moveable points, for the introduction of black lead pencils or ink points, beam compasses for taking large distances, proportional compasses, &c., the uses of which are easily learned. (See *Heather's Treatise on Mathematical Instruments.*)

PLOTTING SCALES.

Plotting scales, also called feather-edged scales, are straight rulers, usually about 12 inches long. Each ruler has scales of equal parts, decimally divided, placed on its edges, which are made sloping, so that the extremities of the strokes marking the divisions lie close to the paper. The primary divisions represent chains, and the subdivisions ten links each, the intermediate links being determined by the eye. Plotting scales may be procured in sets, each with a different number of chains to the inch. They are usually made of ivory or box, and each provided with a small scale called an offset scale for laying down the offsets. In using these scales, the first division or zero, on the plotting scale, is placed coincident with the beginning of the line to be plotted, and so as just to touch that line with the feather-edge: the end of the offset scale is then placed in contact with the edge of the plotting scale; and thus the offsets may be expeditiously pricked off: for which purpose an instrument called a pricker is used, but a hard black lead

pencil, with a fine point, is greatly to be preferred, as it does not injure the paper.

The vernier scale and protractor, the uses of which will be readily understood, are best adapted to laying down extensive surveys, where great accuracy is required.

PLANNING SURVEYS.

In planning or plotting surveys, the upper part of the paper or book, on which the plan is made, should always, if possible, be the north. The chain lines, buildings, fences, &c., ought first to be drawn with a fine black lead pencil: the first should then be dotted with ink, and the latter neatly drawn. Great care is required in the construction of the plan, when the dimensions are to be measured therefrom with the scale. The scale should never be more than three chains to an inch, for when the parts of a plan are large, the dimensions may be taken with greater accuracy. After having found the content of the field or fields, &c., of which any plan consists, it may be laid down by any scale to give it a more convenient size.

THE FIELD BOOK.

The method now generally adopted in setting down field notes, and which has long been found to be the best in practice, is to begin at the bottom of the page and write upwards.

Each page of the book is usually divided into three columns. The middle column is for distances measured on the chain line, at which hedges are crossed, or offsets, stations, or other marks are made; and in the right and left columns, those offsets, marks, and any other necessary observations thereon, must be entered, accordingly as they are situated on the right or left of the chain-line.

The crossing of roads, rivers, hedges, &c., are, by some surveyors, shewn in the field book, by lines drawn across the middle column at the distances where they are crossed, and by others these crossings are shewn by lines drawn on part of the right and left hand columns, opposite the distances where they are crossed by the chain line; and buildings, turns of fences, corners of fields, to which offsets are taken, are usually shewn by lines sketched in a similar situation to the middle column, as the fences, buildings, &c., have to the chain line. Thus a representation of the chief objects in the survey may be sketched in the field, which will give essential assistance in laying down the plan. The stations are usually numbered, for the sake

of reference, and marked thus ⊙. The bearing of the first main line is usually taken by surveyors, from which the position of the plan with respect to the north is determined. This may be done by a common pocket compass, where great accuracy is not required.

R. of ⊙ 2, and L. of ⊙ 5, &c., denote that the following lines are measured to the right of station 2, and to the left of station 5, respectively.

To Survey with the Chain and Cross.

An acre of land is equal to 10 square chains, that is 10 chains in length and one in breadth, or 1000 links in length and 100 in breadth; an acre, therefore, contains 100,000 square links, as per table of square measure below. Hence the contents in square links are, in the following examples, divided by 100,000, or what is the same thing 5 figures to the right are cut off for decimals, the figures remaining on the left being acres. The decimals are then multiplied by 4 for rods, and again by 40 for poles.

The following tables exhibit the number of chains and links in the different units of lineal measure, and the number of square chains and links in the different units of square measure.

A TABLE OF LINEAR MEASURE.

Links.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Mile.
25	16½	5½	1			
100	66	22	4	1		
1,000	660	220	40	10	1	
8,000	5,280	1,760	320	80	8	1

A TABLE OF SQUARE MEASURE.

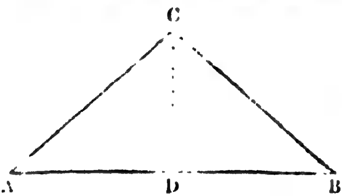
Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Poles, or Perches	Sq. Cha.	Rods.	Acres.	Sq. Mile.
625	272¼	30¼	1				
10,000	4,356	484	16	1			
25,000	10,890	1,210	40	2½	1		
100,000	43,560	4,840	160	10	4	1	
64,000,000	2,787,400	3,097,600	102,400	6,400	2,560	640	1

Problem I.

TRIANGULAR FIELDS.

1. Let A B C be a triangle, of which the survey, plan, and content are required.

Set up poles or marks at the angles A, B, and C, and measure from A towards B, and when at or near D, try with the cross for the place of the perpendicular C D; plant the cross, and turn it till the marks A and B can be seen through one of the grooves; then look through the other groove, and, if the mark at C can be seen through it, the cross is in the right place for the perpendicular; if not, move the cross backward or forward till the three marks can be seen as before directed. Suppose the distance A D to be 625 links, and the whole A B, to be 1257 links; return to D, and measure the perpendicular D C, which suppose to be 628 links, thus completing the survey of the triangle.



CONSTRUCTION.

From a scale of equal parts, or plotting scale, lay off the base A B = 1257 links; on which take A D = 625 links; at D erect the perpendicular D C, which make = 628 links; join A C, C B, then A B C is the plan of the triangle.

TO FIND THE CONTENT.

Rule.—Multiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

1. The dimensions being the same as found above, required the content.

Ans. $1257 \times 628 \div 2 = 3{\cdot}94698$ acres $= 3a.\ 3r.\ 32p.$

2. The distance from the beginning of the base to the place of the perpendicular is 375 links, the whole base 954, and the perpendicular 246; what is the area of the triangle.

$954 \times 246 \div 2 = 1{\cdot}17342 = 1a.\ 0r.\ 27\frac{1}{4}p.$ the content.

3. Measuring the base of a triangle the place of the perpendicular was found at 863 links, and its length 645; the whole base was 1434 links; required the plan and area.

Area. $4a.\ 2r.\ 20p.$

Problem II.

FIELDS IN THE FORM OF TRAPEZIUMS.

Fields in this form are usually divided into two triangles by a diagonal, which is a base to both the triangles.

Let A B C D be a field in the form of a trapezium, the plan and area of which is required.

Measure from A towards C; and let the place of the perpendicular *m* B be at 5·52, and its length 3·76, also let the place of the perpendicular *n* D be at 11·82, and its length 3·44, and the length of the whole diagonal A C be 13·91 chains, which completes the survey: but it is usual also to measure the other diagonal B D for a proof line, which is found to be 9·56 chains.

Note 1. The construction of each of the two triangles, forming the trapezium, is the same as the construction given to the first example in Prob. I.

Note 2. The longer of the two diagonals should always be selected for the base of the two triangles forming the trapezium, for sometimes the perpendicular will not fall on the shorter diagonal, without its being prolonged; and when this is the case with both diagonals, one of the sides may be taken for a base, or two of the sides, if necessary.

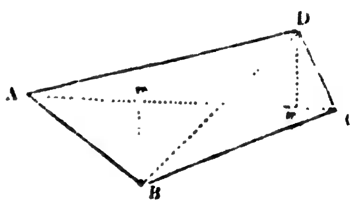
TO FIND THE CONTENT.

Rule.—Multiply the sum of the two perpendiculars by the diagonal, and half the product will be the content.

EXAMPLES.

1. Let the measurement of a trapezium be as above found; required the content.

```
      344
      376
     ----
      720
     1391
     ----
    27820
    9737
   ------
2) 1001520
   -------
   500760
        4
   -------
   003040
       40
   -------
   121600
```



Ans. 5*a.* 0*r.* 1*p.*

2. From the following notes, plan and find the content of a field.

Perpendiculars on left.	Base or Station Line.	Perpendiculars on right.
	to ⊙ C	
	3250	
	2504	1046 D
B 1278	1272	
Begin	at ⊙ A	and range West.

Content. 37*a*. 3*r*. 2*p*.

3. Give the plan and area of a field from the following notes.

	A C	
	872	
B 652	731	
	423	545 C
Begin at	⊙ A	and range East.

Area. 5*a*. 0*r*. 35*p*.

ANOTHER METHOD.

A four-sided field may frequently be surveyed by dividing it into two triangles and a trapezoid, by perpendiculars on its longest side.

TO FIND THE CONTENT.

Rule.—Multiply the sum of the two perpendiculars by their distance on the base line for the double area. The double areas of the two triangles must be found as in Prob. I., and both be added to the double area of the trapezoid; the sum being divided by 2, will give the content required.

EXAMPLES.

1. Required the survey and area of the following field.

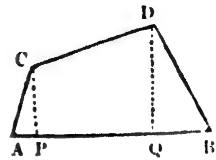
Measure the base A B, and put down in the field book the distances of P and Q, where the perpendiculars rise, &c., as follows :—

	to ⊙ D.
	AB=1110
QD=595	AQ= 745
PC=352	AP= 110
Perpen.	From ⊙ A go E.

Triangle ACP
352
110
38720

Triangle QDB
595
352
9975
3570
1785
217175

Trapezoid PCDQ
352 }
595 } perp.
947 sum.
365 = PQ
4735
2841
5682
601345
38720
217175
2)857240
4·2862
4
1·1448
40
5·7920



Area. 4*a*. 1*r*. 5¾*p*.

2. Required the plans and areas of two fields from the following notes.

	A B	
	1169	
E	615	339 D
G	234	461 E
From	⊙ A go	W.

	A B	
	1448	
D 513	1102	E
C 683	436	G
From	⊙ A	go E.

Problem III.

TO SURVEY FIELDS CONTAINED BY MORE THAN FOUR SIDES.

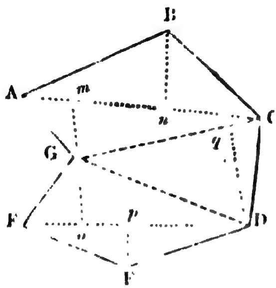
Fields or plots of ground bounded by more than four sides, may be surveyed by dividing them into trapeziums and triangles.—Thus, a field of five sides may be divided into a trapepezium and a triangle; of six sides, into two trapeziums; of seven sides into two trapeziums and a triangle; &c.

TO FIND THE CONTENT.

Rule.—By the two last Problems, find the double areas of each trapezium and triangle in the field; add all the double areas together, and half their sum will be the content.

EXAMPLES.

1. Lay down a field and find its area from the following dimensions.



	to ⊙ D	
	520	
p	288	80 E
G 120	206	o
	Go to ⊙ F	
	to ⊙ G	
	440	
D 230	152	q
	L. of ⊙ C	
	to ⊙ C	
	550	
B 180	410	n
m	135	130 G
Begin	at ⊙ A	range E.

CONSTRUCTION.

The above field is divided into two trapeziums A B C G G D E F and the triangle G C D. Draw the diagonal A C which make = 550 links; at 135 and 410 set off the perpendiculars mC = 130, and nB = 180 links respectively; join A B B C, C G, and G A, and the first trapezium will be completed Then on C G, lay off Cq = 152, and draw the perpendicular qD = 230; join C D, D G, and the triangle is completed Lastly, with centre G and radius oG = 120 describe an arc and with centre D and radius oD = 314 (= 520 − 206) describe another arc, intersecting the former in o: through o draw the diagonal D F = 520 links, upon which, at 288 links, draw the perpendicular pE; join D E, E F, F G, and the figure will be completed.

```
 130        440       120     Double areas.
 180        230        80        170500 trap. A B C G
 ---        ---       ---        101200 tri.   C D G
 310      13200       200        104000 trap. D E F G
 550        880       520      ---------
-----     ------    ------    2)3·75700
15500     101200    104000    ---------
1550                            1·87850 — 1a. 3r. 20¼p.
------
170500
```

2. Required the plan and areas of two fields from the following dimensions.

First Field.

	to ⊙ A	
	504	Base.
E 98	233	
Return	to ⊙ B	
	to ⊙ D	
	673	Diag.
C 207	472	
	427	268 B
Begin	at ⊙ A	range W.

Area. *1a. 3r. 15p.*

Second Field.

	to ⊙ F	
	970	Diag.
	520	181 B
E 290	413	
	R. of ⊙ D	
	to ⊙ D	
	744	Diag.
C 161	386	
	303	333 B
Begin	at ⊙ A	range W

Area. *4a. 0r. 9½p.*

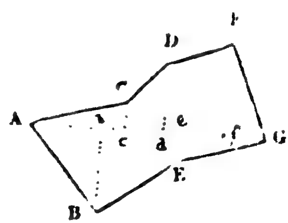
ANOTHER METHOD.

A small piece of land, having several sides, may sometimes be most conveniently measured by taking one diagonal, and upon it erecting perpendiculars to all the angles on each side of it. The piece will thus be divided into right angled triangles and trapezoids, the areas of which must be calculated as in the two last Problems.

EXAMPLES.

1. Required the plan and area of a field from the following notes.

	to ⊙ G	
	1020	
F 470	890	*f*
e	610	50 E
D 320	585	*d*
C 70	440	*c*
b	315	350 B
Begin	at ⊙ A	go E.



NOTE. The method of planning the above field is sufficiently clear, from the preceding field-notes and from what has been already done.

Triangle A C *c*.	Trape. C *c d* D.	Trape. D *d f* F.	Tri. F *f* G.	Tri. A *b* B.
A *c* = 440	D *d* = 320	D *d* = 320	*f* G = 130	A *b* = 315
C *c* = 70	C *c* = 70	F *f* = 470	F *f* = 470	B *b* = 350
30800	sum = 790	sum = 790	9100	15750
	c d = 145	*e f* = 395	52	1545
	13050	3950	61100	170250
	435	2370		
	56550	240950		

Trapezoid B *b e* E.	Triangle E *e* G.	Double areas.
		30800
		56550
B *b* = 350	G *e* = 410	240950
E *e* = 50	E *e* = 50	61100
		170250
sum = 400	20500	118000
b e = 295		20500
118000		2)698150
		3·49075

= 3*a*. 1*r*. 38½*p*. Area.

Problem IV.

FIELDS INCLUDED BY ANY NUMBER OF CROOKED OR CURVED SIDES.

When a field or estate is bounded by crooked fences, a line must be measured as near to each of them, as the angles or bends will permit; in doing which an offset must be taken to each corner or bend in the fence. The offsets or perpendiculars thus erected, will divide the whole offset space into right angled triangles and trapezoids, the areas of which may be found as already shewn.

Note 1. When the offsets are short, that is, not greatly exceeding a chain in length, their places on the line may be found by laying the offset-staff at right angles to the chain, as nearly as can be judged by the eye; but when the offsets are large, and correctness is required, their places must be found by the cross, and measured by the chain.

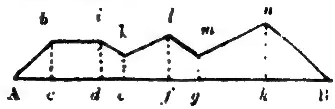
Note 2. The quickest method of laying down offsets, is, by laying the feather-edge of the plotting scale against the base or chain line, and sliding the offset scale along the feather-edge to the several distances of the offsets, and pricking off their lengths, corresponding to their several distances.

NOTE 3. Unskilful surveyors usually add all the offsets taken on one line together and divide the sum by their number for a mean breadth; but this method is very erroneous, especially where the offsets vary greatly in length, and should therefore be avoided where great accuracy is required.

EXAMPLES.

1. Required the plan and content of a right-lined piece of ground by offsets, from the following notes.

	to ⊙ B	
o	955	
n 91	785	h
m 57	634	g
l 88	510	f
k 70	340	e
i 84	220	d
h 62	45	c
o	00	
Begin	at ⊙ A	range E.



$A\,c = 45$
$c\,h = 62$
90
270
2790

$c\,h = 62$
$d\,i = 84$
146
$c\,d = 175$
730
1022
146
25550

$d\,i = 84$
$e\,k = 70$
154
$d\,e = 120$
18400

$e\,k = 70$
$f\,l = 88$
158
$e\,f = 170$
11060
158
26860

$f\,l = 88$
$g\,m = 57$
145
$f\,g = 124$
580
290
145
17980

$g\,m = 57$
$h\,n = 91$
148
$g\,h = 151$
148
740
148
22348

$h\,B = 170$
$h\,n = 91$
170
1530
15470

Double areas.
2790
25550
18480
26860
17980
22348
15470
2) 1·29478
0·64739 = 0*a*. 2*r*. 23*p*.

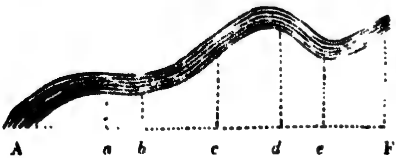
Calculation by the erroneous method (See Note 3).

00	955	= A B.
62	$56\frac{1}{2}$	= mean breadth.
84		
70	5730	
88	4775	
57	477	
91		
00	0·53957	= 0*a*. 2*r*. 6*p*. Content by this method, *which is 17 perches too little.* For this method is always erroneous except when the offsets stand at equal distances from one another, and when the first and last offsets are both 0.
8) 452		
$56\frac{1}{2}$		

Some omit all the offsets that are 0, dividing the sum of the offsets by the number of real offsets; by this method we shall have

6) 452	955
$75\frac{1}{3}$	$75\frac{1}{3}$
	4775
	6685
	318
	0·71947 = 0*a*. 2. 34*p*., *which is 11 poles too much.*

2. To lay down a crooked piece of land, adjoining a river from the following notes.



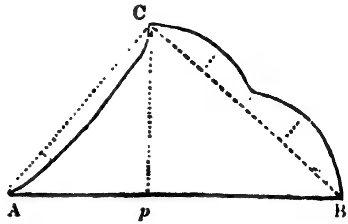
	to ⊙ F	
197	720	
163	600	
226	500	
139	350	
80	200	
74	100	
0	000	[E.
Begin	at ⊙ A	range

The content is found by the same method as in the preceding example.

3. Plan and find the area of a field from the subjoined notes.

	to ⊙ A
0	480
37	350
28	160
0	000
	L. of ⊙ C

to ⊙ C	
585	0
450	57
320	40
200	72
100	47
000	0
L. of ⊙ B	



	to ⊙ B	
	743	
C 382	290	p
From	⊙ A	go E.

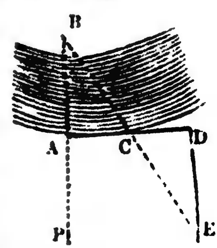
Having found the area of the triangle A B C, the areas of the offsets on the line B C must be added thereto, and the sum of the areas of the *insets* on the line C A must be subtracted from the sum, and the remainder will be the content of the field.

NOTE. The area of the triangle A B C may be found, when the measurement of all three sides are given, (which is the case in the present example,) either by calculation, as shall hereafter be shewn, or by measuring the perpendicular from the plan, which, as already shewn, may be laid down from the three sides of the main triangle.—The areas of fields, having a great number of crooked and curved fences, are found by the method given in the last Example, page 51.

PROBLEM V.

TO MEASURE A LINE ACROSS A WIDE RIVER.

Let the annexed figure be a river, which is required to be crossed by the chain line P B. Fix, or cause to be fixed, a pole or mark at B, at or near the margin of the river, in the line to be measured; erect the perpendicular A D, measuring A C and C D of any equal lengths; at D erect the perpendicular D E; on arriving at E, in the direction B C, the distance D E will be equal to A B, the required breadth of the river.



From the arrangement of the lines in the figure, it is evident that the triangles C A B, C D E are equiangular, and since A C was made = C D, the triangles are equal in all respects, and consequently A B = D E.

NOTE 1. For various other methods of measuring obstructed lines, under different circumstances, see *Baker's Land and Engineering Surveying*, Chap. III.

NOTE 2. A sufficient detail of methods of surveying by the help of the cross, which, though not much used by experienced surveyors, is a simple instrument, and its use readily understood by students. This method is, therefore, a proper introduction to the higher branches of surveying; besides, in rural districts, villages, &c., few surveyors use the more expensive instrument, the chain and cross being found quite sufficient to measure the quantities of growing crops, and other such small surveys as may be there required.

LAND SURVEYING BY THE CHAIN ONLY.

This method of surveying has long been adopted by experienced surveyors, who have found it, in general, more accurate and expeditious, as well as better adapted to laying down extensive surveys especially where no serious obstructions from woodlands, water, buildings, &c., exist: the use of the cross, in this method, being entirely excluded by some surveyors, and by others only used for secondary purposes, as for taking occasionally long offsets, or for squaring of lines obstructed by buildings, water, &c. Instead of the cross some use the optical square for these purposes; while some erect perpendiculars with the chain only. See Chap. III., *Baker's Land and Engineering Surveying.*

PROBLEM I.

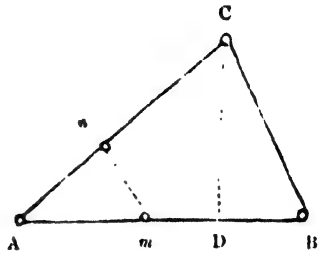
TRIANGULAR FIELDS.

When a triangular field, or piece of ground in that shape, is to be surveyed, set up poles or marks at each corner, and measure each side, leaving marks in at least two of the lines, and entering their distances in the field book; then measure the distance between the two marks for a proof line:—or, one mark only may be left in one of the lines, which may be connected with its opposite angle for a proof line.

EXAMPLES.

1. Required the construction and area of a field from the following dimensions.

	to ⊙ *n*	
Proof	384	line.
From	⊙ *m*	
	to ⊙ A	
	1244	
⊙ *n*	700	
	L ⊙ C	
	to ⊙ C	
	852	
	L ⊙ B	
	to ⊙ B	
	1338	
	1000	
⊙ *m*	600	
From	⊙ A	range E.



When the triangle A B C is constructed, the proof line *m n* will be found to measure 384 links, shewing that there has been no error in the work: also the perpendicular C D will be found to be 770 links; whence the area of the triangle $= 1338 \times 770 \div 2 = 5 \cdot 15130$ $= 5a.\ 0r.\ 24p.$ the area.

NOTE. If the proof line measured from the plan, does not exactly, or very nearly, agree with that measured in the field, some error has been made, and the work must be repeated.

TO FIND THE AREA OF A TRIANGLE FROM THE THREE SIDES.

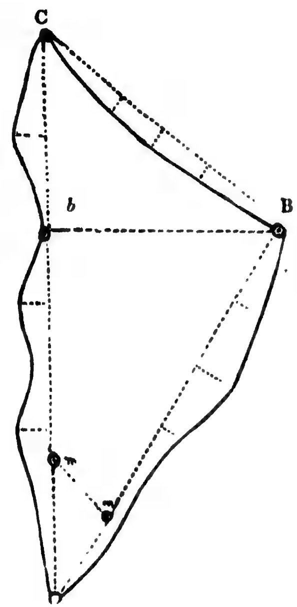
RULE.—From half the sum of the three sides subtract each side severally and reserve the three remainders; multiply the half sum continually by the three remainders, and the square root of the product will be the area.

NOTE. By this rule the area of a triangle may be found without laying it down, or finding the perpendicular.

Adopting the preceding example, we have by the rule,
$$\frac{1338 + 852 + 1244}{2} = 1717 = \text{half sum of the three sides.}$$
Then $1717 - 1338 = 379 = $ 1st remainder; $1717 - 852 = 865 = $ 2nd remainder; $1717 - 1244 = 473 = $ 3rd remainder; whence $\sqrt{(1717 \times 379 \times 865 \times 473)} = 5 \cdot 15022 = 5a.\ 0r.\ 24p.$ the sum as the area already found by measuring the perpendicular from the plan.

NOTE. This method of finding the areas of triangles is very little used in practice, on account of its requiring a tedious calculation, which may, however, be more readily performed by logarithms.

2. It is required to lay down a survey and find its content from the following field notes.



	to ⊙ A	
	2504	0
	2000	74
⊙ *m*	1860	351 to ⊙
	1650	137
	1430	90
	1220	144
	850	30
	425	110
	000	0
	L. ⊙ C	
	to ⊙ C	
0	1346	
80	1072	
128	708	
98	458	
0	000	
	L. ⊙ B	
	to ⊙ B	
	1946	0
	1490	96
	1200	152
	1000	112
⊙ *n*	600	
	520	50
	000	
From	⊙ A	go N. E.

Having drawn the figure, the proof line *m n* will be found to measure 351 links, as in the field notes; and the perpendicular B *b* to be 1056 links.

Double areas.

2644224	Triangle A B C
655676	Offsets on A B and A C
3299900	Sum
200616	Insets on B C
2)3099284	Difference
15·49642	= 15*a*. 2*r*. 0*p*. nearly, the area required.

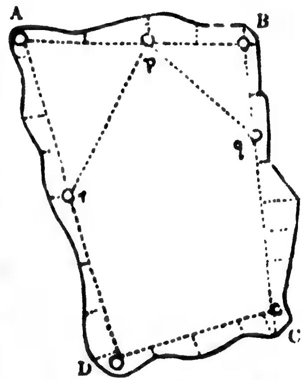
Problem II.

FOUR SIDED FIELDS.

When a field has four sides, straight or crooked, measure the four sides, or lines near them, if crooked, taking the offsets: also measure one or both the diagonals, one of which will serve as a base in plotting the work, and the other for a proof-line; or the proof-line may be measured in any other direction that may be most convenient.

Sometimes the measurement of both the diagonals is prevented by obstructions, in such cases it will be sufficient to measure tie-lines across two of the angles of the trapezium, at the distance of from two to five chains from each angle, according to the size of the field. These tie lines with their distances from the angles on the main lines will be found sufficient for planning the lines and proving them.

EXAMPLE.



In the annexed figure the lines A B, B C, C D, D A are measured, marks being left at p, q, and r, and their respective distances on the lines noted in the field book, thus furnishing the following method of laying down the plan.

On A B, as a base, take A p = given distance, and with the distances A r, p r, and centres A and p describe arcs cutting in r; then prolong A r, and lay off thereon the given length A D. In the same manner construct the triangle p B q, and make B C = its given length. Lastly, join D C, which must be of the length shewn in the field book, otherwise there has been some mistake either in the measurement, or in laying it down. Should this be the case, the whole of the work, firstly on the plan, and secondly in the field, must be gone over again till the error be discovered.

Problem III.

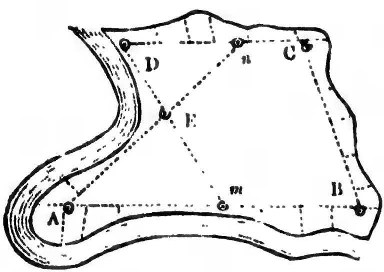
FIELDS HAVING MORE THAN FOUR SIDES.

Various methods will suggest themselves to the surveyor for taking lines to lay down a field that requires more than four main lines to take its boundary. The method of dividing such

fields into trapeziums and triangles is, in most cases, circuitous, and displays little skill on the part of the surveyor, especially where all the sides are crooked, and where a plan is required. A few methods of surveying fields of this kind will, therefore, be presented to direct the student; although their variety of shape is so endless, that no general rule can be given for laying out lines on the ground, that shall give an incontestably accurate plan. To tie every angle in succession, though true in principle, is by no means a safe method, especially where there are a great number of angles to be tied, as an error in one of the tie-lines will derange the whole of the work, without affording the means of detecting where the error lies.

NOTE. The following example of surveys of this kind occurred in part of the author's extensive practice, as a surveyor of parishes under the Tithe-Commissions. The student is recommended to sketch the following specimens on a large scale, and find their contents by the usual methods.

EXAMPLE.



Here a field of five sides is surveyed by the same number of lines, viz. A B, B C, C D, D m, and A n, the last two intersecting in E. These lines evidently constitute a decisive proof among themselves, and all of them are available in taking the boundary.

In surveying this field (poles or natural marks being supposed to be fixed at A, B, C, D, and E) commence close to the river's edge, in the line A B prolonged backwards, enter the offsets and the station A in the field-book. On arriving at ⊙ m, in the direction E D, enter its distance, and so on to ⊙ B, measuring the line to the fence; from B proceed to C, in like manner, measuring beyond the station to the fence. The place of the ⊙ n is to be noted, on arriving in the direction E A, while measuring C D. D m is next measured, the place of the ⊙ E being noted. Lastly, go from m to A, and measure A n, entering the place of the ⊙ E a second time, all the offsets being supposed to be taken during the operation.

Construction of the plan. Select the distances A m, A E, and E m from the field-book, and with them construct the triangle A m E, prolong the sides to their entire lengths, up to the

boundaries, and fix the places of the stations B, *n*, and D. Now, if the measured length of D *n* just fit between D and *n*, the work is right with respect to the triangles A E *m*, E D *n*. Lastly, prolong D *n* to the ⊙ C, and, if the distance from thence to the ⊙ B be the same as shewn by the field book, the whole of the work is right. But, if the distance D *n* do not agree, the work must be examined from the beginning; if only the distance B C fail, then only that distance and the portions *m* B, C *n* need be examined.

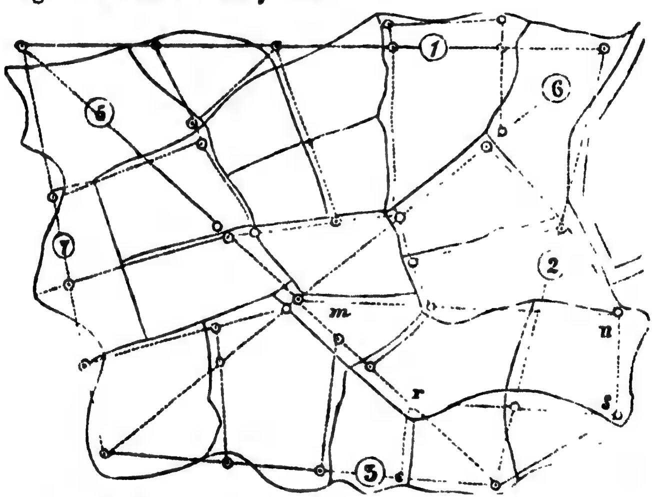
NOTE. For a variety of other methods of surveying irregular fields, small estates, &c., &c., see *Baker's Land and Engineering Surveying*, Chap. III.

PROBLEM IV.

SURVEYING LARGE ESTATES OR PARISHES BY THE CHAIN ONLY.

Having perambulated the boundary of the estate, parish, or lordship to be surveyed, if you find that its boundary approaches somewhat near to that of a four-sided figure, or trapezium, the system of fundamental lines, adopted by order of the Tithe Commissioners of England and Wales, is to be preferred. These fundamental lines are six in number, of which four must run close by, or as nearly as possible to, the boundary in question, thus forming a trapezium, four lofty station poles being placed at each angle, as objects for running the lines; the other two lines must form the diagonals of this trapezium, and therefore pass through the central parts of the survey, intersecting each other, the points of intersection being noted on measuring each line, so that when the system of lines are laid down on the plan, the proof of the accuracy of the work may be fully established, before the minor operations, or filling up, as it is called, is commenced. It will be necessary, moreover, in almost every case, to range the lines between every two of the main stations with long slender ranging poles, as the intervention of hills, fences, trees, buildings, &c., will frequently interrupt the view of even the loftiest station poles that can be obtained; and more especially so, when the main stations are at a great distance, which depends on the magnitude of the survey, and is sometimes as much as ten miles. In measuring these main lines, every fence, road, stream, building, &c., which is passed or crossed must be noted in the field book, the several crossings and bends being sketched therein, to the latter of which offsets must be taken. Stations must also be left on these main lines, at convenient situations for taking the interior fences, &c., of the survey, and their distances carefully noted in the field book. From and to the sta-

tions, thus left, or from and to points near them, secondary lines must be run, as near the interior parts of the survey as possible, the crossings, offsets, and other remarks being made in the field book, as already directed for the measurement of the main lines. These secondary lines will accurately fit between the points from and to which they have been measured, when laid down on the plan; thus forming a net work of small triangles within the four large triangles, into which the survey is divided by the six fundamental lines. This principle of proof is founded on the obvious property of triangles having a common angle always fitting one within another, the common angle of both being coincident. The lines marked with the figures 1 to 6, represent the system in question, those without figures are the secondary lines.



The main lines are numbered with the figures 1, 2, 3, &c., in small circles, as the most convenient method of reference to the field book: the secondary lines must have these numbers continued on them, for the same purpose, but this is not done in the diagram, to avoid confusing it.

It will be seen that the secondary lines *m n*, *r s* are prolonged beyond the system of main lines, to give stability to the parts of the survey that protrude beyond line 2.

NOTE. For the method of keeping the field book in extensive surveys, the description and use of the theodolite and other surveying instruments, see [illegible] *Land and Engineering* [illegible]

ENGINEERING SURVEYING.

LEVELLING.—DEFINITION OF LEVELLING.

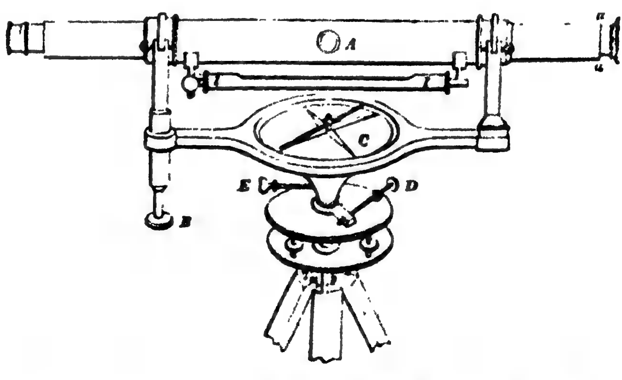
By the art of levelling the inequalities of the upper boundary of any section of the earth's surface may be shewn, and thence may be determined the several heights of any number of points in that boundary, above or below an assumed line, called a level line; though, in reality, this line is a great circle of the earth, and is such as would be derived from a section of the surface of still water.

LEVELLING INSTRUMENTS.

(1.) Levelling instruments all depend on the action of gravity: of these the plumb-line, on which the mason's level depends, is the most simple; but it cannot be used in extensive operations, on account of its practical inconvenience. The fluid, or water level, in all its modifications, is also found inconvenient for extensive practice.

(2.) Spirit levels are now commonly used, as the most accurate instruments for finding the differences of level, or vertical distances between two stations: of these there are several, we shall only here describe the Y level.

THE Y LEVEL.



The foregoing figure represents this instrument. A is an achromatic telescope, resting on two supporters, which in shape resemble the letter Y; hence the name of the instrument. The lower ends of these supporters are let perpendicularly in a

strong brass bar, which carries a compass box C. This compass is convenient for taking bearings, and has a contrivance for throwing the needle off its centre, when not in use. One of the Y supporters is fitted into a socket, and can be raised or lowered by the screw B.

Beneath the compass box, which is generally of one piece with the bar, is a conical axis passing through the upper of two parallel plates, and terminating in a ball supported by a socket. Immediately above the upper parallel plate is a collar, which can be made to embrace the conical axis tightly by turning the clamping screw E; and a slow horizontal motion can be given to the instrument by means of the tangent screw D. The two parallel plates are connected together by the ball and socket already mentioned, and are set firm by four mill-headed screws, which turn in sockets fixed to the lower plate, while their heads press against the under side of the upper plate, and thus serve the purpose of setting the instrument truly level.

Beneath the lower parallel plate is a female screw, adapted to the staff head, which is connected with brass joints to three mahogany legs, which support the instrument.

The spirit level *l l* is fixed to the telescope by a joint at one end, and a capstain headed screw at the other, to raise or depress it for adjustment.

(3). Previous to using this instrument the following adjustments must be attended to.

1. *The adjustments of the telescope for parallax and collimation.*

2. *The adjustment of the bubble tube.*

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*

1. *The adjustment for parallax and collimation.* Move the object-glass by the screw, and the eye-glass with the hand, till distant objects and the cross wires within the telescope, appear clearly defined; and the adjustment for parallax will be completed. Next, direct the telescope to some well-defined object at a great distance; and see that the intersection of the cross wires cut it accurately; then loose the clips that confine the telescope in the Ys, and turn it round on its axis, observing whether the centre of the wires still continue to cut the object, during a whole revolution. If it does, it is in adjustment; if not, the line of collimation, or optical axis of the instrument, is not in the line joining the centres of the eye and object-glasses. *To correct this error,* turn the telescope on its axis, and by

means of the four conjugute screws *a, a*, &c., that move the cross wires, correct for half the error, alternately loosing one screw and tightening its opposite one, till the cross wires cut the same point of the distant object, during an entire revolution of the telescope round its axis.

2. *The adjustment of the bubble tube.*—Move the telescope till it lies in the direction of two of the parallel plate screws, and by giving motion to these screws bring the air-bubble to the centre of its run. Now reverse the telescope carefully in the Ys, that is, change the places of its ends; and should the bubble not settle in the same point of the tube as before, it shews that the bubble tube is out of adjustment, and requires correcting. The end to which the bubble retires must then be noticed, and the bubble made to return one-half the distance by turning the parallel plate screws, and the other half by turning the capstain headed screws at the end of the bubble tube. The telescope must now again be reversed, and the operation repeated, until the bubble settles at the same point of the tube, in the centre of its run, in both positions of the instrument. The adjustment is then perfect, and the clips, that confine the telescope in the Ys should be made fast.

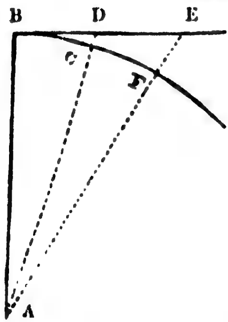
3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*—Place the telescope over two of the parallel plate screws, and move them, unscrewing one while screwing up the other, until the bubble of the level settles in the centre of its run; then turn the instrument half round on its vertical axis, so that the contrary ends of the telescope may be over the same two screws, and, if the bubble does not again settle in the same point as before, half the error must be corrected by turning the screw B, and the other half by turning the two parallel plate screws, over which the telescope is placed. Next turn the telescope a quarter round, that it may be over the other two screws, and repeat the same process with these two screws; and when, after a few trials, the bubble maintains the same position in the centre of its run, while the telescope is turned round on the vertical axis, this axis will be truly vertical; and the axis of the telescope being horizontal, by reason of the previous adjustment of the bubble tube, will be perpendicular to the vertical axis, and remain truly horizontal, while the telescope is turned completely round. The adjustment is therefore perfect.

There are several other highly approved levelling instruments, as Troughton's and Gravatt's levels, &c., for the descriptions of which see *Baker's Land and Engineering Surveying.*

LEVELLING STAVES.

(6.) The best constructed levelling staff (Gravatt's) consists of three parts sliding one within another, and, when opened out for use, form a staff 17 feet long, jointed together something after the manner of a fishing rod. The whole length is divided into hundredths of a foot, alternately coloured black and white, and occupying half the breadth of the staff; but for distinctness the lines denoting tenths of feet are continued the whole breadth, every half foot or five tenths being distinguished by a conspicuous black dot on each side, the whole feet being numbered with the figures 1, 2, 3, &c.

CORRECTION FOR CURVATURE.



(7.) Let B D E be a horizontal line, that is, such as would be given by the line of sight of a level, properly adjusted; B C F an arc of a great circle of the earth, and A its centre. It will at once appear from the figure, that the heights D C, E F, of the apparent level B E, above the true level increase successively from the point B. The height E F of the apparent level above the true, is equal to the square of the distance B E divided by twice the earth's radius A B, that is $E F = \frac{B E^2}{2 A B}$,* and similarly $D C = \frac{B D^2}{2 A B}$, &c., therefore the corrections for curvature, D C, E F, &c., vary as the squares of the distances B D, B E, &c., since 2 A B is a constant quantity.

Taking the earth's radius to be 3979 miles, and assuming the distance B D to be 1 mile, then the correction for curvature $D C = B D^2 \div 2 A B = 1^2 \div 7958 = \frac{1}{7958}$ of a mile $= 7{\cdot}962$ inches = nearly 8 inches. If the distance B E = 3 miles, then the correction $E F = B E^2 \div 2 A B = \frac{9}{7958} = 71{\cdot}658$ inches, or nearly 6 feet.

Let any distance B D = d in miles, and the correction for curvature for 1 mile be taken = 8 inches = $\frac{2}{3}$ of a foot, which it is very nearly; then

$$\text{correction} = \frac{2\,d^2}{3} \text{ feet,}$$

for any distance d in miles:

* The demonstration of this formula is given in my edition of *Nesbit's Surveying*, p. 348.

and let $\frac{c}{80} = d$, c being chains; then

$$\text{correction} = \frac{2\,d^2}{3} = \frac{2 \times 12\,c^2}{3 \times 80^2} = \frac{c^2}{800} \text{ inches.}$$

for any distance c in chains.

CORRECTION FOR REFRACTION.

(8.) The effect of the earth's curvature is modified by another cause, arising from optical deception, namely, refraction; the correction for which varies with the state of the atmosphere, but it may generally be taken at $\frac{1}{7}$ of the correction for curvature, as an average; and since refraction makes objects appear higher than they really are, the correction for it must be deducted from that for curvature.

EXAMPLES.

1. Required the correction for curvature and refraction, when the distance of the object is $2\frac{1}{2}$ miles.

$$\frac{2}{3} \times (2{\cdot}5^2 \quad \frac{2 \times 6{\cdot}25}{3} = 4{\cdot}166 \text{ cor. for curvature.}$$

$\frac{1}{7}$ of which is............ ·693 cor. for refraction.

Difference................ 3·473 feet, cor. required.

2. Required the correction, as in the last example, when the distance is 60 chains.

$60^2 \div 800 = 4{\cdot}5$ cor. for curvature.

$\frac{1}{7}$ of which is .. ·643 cor. for refraction.

Difference......3·857 inches, cor. required.

3. From a point in the Folkstone road, the top of the keep of Dover Castle was observed to coincide with the horizontal wire of a levelling telescope, when adjusted for observation, and therefore was apparently on the same level; the distance of the instrument from the castle was $4\frac{1}{2}$ miles, required the correction for curvature and refraction, that is, the true height of the keep of the castle above the point of observation.

$$\frac{2}{3} \times (4{\cdot}5)^2 = \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ feet, cor. for curvature}$$

$\frac{1}{7}$ of which............= 1·93 feet, cor. for refraction.

Difference ....= 11·57 feet, cor. required.

See also the tables for these corrections at the end of the book.

PRINCIPLES AND PRACTICE OF LEVELLING.

To find the differences of the levels of several points on the surface of the earth.

(7.) Before entering on this subject, it will be proper to state that the corrections for curvature and refraction, already explained, are seldom applied in the practice of levelling, the spirit level being usually placed midway between the stations, the levels of which are to be observed, hence the resulting corrections for each station are equal, and therefore the difference of the levels at the two stations is as truly shewn by the difference of the readings of the two staves, fixed thereon, as if the corrections had been made. Thus the trouble of making these corrections is avoided by *simply placing the instrument midway between the two staves.*

(8.) Let it be required to find the difference of level between the points A and G. A levelling staff is erected at A, the instrument is set up and adjusted at B, another staff is also erected at C, at the same distance from B that B is from A, as nearly as can be judged by the eye; the reading of the two staves are

then noted; the horizontal lines, connecting the staves with the instrument, represent the visual ray or level line of sight. The instrument is then conveyed to D, and the staff that stood at A is now removed to E, the staff C retaining its former position, only its graduated side turned to the instrument, and from being the fore staff at the last observation, it is now the back staff; the reading of the two staves are again noted, and the instrument removed to F, and the staff C to the point G, the staff at E retaining its position, now in its turn becomes the back staff, and so on to the end of the work, which may thus be continued to any extent. The difference of the readings of the staves at A and C will shew the difference of level between the points or stations A and C, because the visual line of the instrument is virtually level, and the same is true with respect to every two consecutive stations.

EXAMPLE.

Back sight on staff A	10·66 feet
Fore sight on staff C	11·78
The fall from A to C	1·12 difference.

Because when the front reading is the greater the ground falls, and *vice versa.*

Back sight on staff C	13·36	
Fore sight on staff E	9·16	
The rise from C to E	4·20	difference.
Subtract the fall from A to C...	1·12	
The rise from A to E	3·08	difference.

Because the rise from C to E is greater than the fall from A to C, their difference shews the total rise.

Back sight on staff E	7·62	
Fore sight on staff G	8·16	
The fall from E to G	0·54	difference.

This fall taken from the rise from A to E, that is,

3·08
0·54

gives the total rise from A to G...............2·54, or nearly 2 feet 6½ inches.

The difference of the sums of the back and fore readings of the staves, will more readily give the difference of level between A and G: thus,

	Back sights.	Fore sights.
	feet.	feet.
	10·66 at A	11·78 at C
	13·36 at C	9·16 at E
	7·62 at E	8·16 at G
sums	31·64	29·10
	29·10	

2·54 difference of level, the same as before.

TO DRAW A SECTIONAL LINE OF SEVERAL POINTS IN THE EARTH'S SURFACE, THE LEVELS OF WHICH HAVE BEEN TAKEN.

Let *a, b, c, d, e, f,* and *g* be the several points; then, in order to draw the section to shew the undulations of the ground between *a* and *g*, the distances of the several points from *a*, in addition to their levels, must be taken; this is usually done during the operation of levelling. These distances, with the back and fore sights, may be arranged in a level book of the following form, which, though not the form practically used, will probably be more clearly understood. *(See Fig. page 124.)*

LEVEL BOOK.

Back Sights.	Fore Sights.	Fall.	Rise.	Reduced Levels.	Distances in Chains, and Remarks.
3·50	5·65	2·15		2·15	4·60 at *b* on road.
4·10	10·85	6·75		8·90	7·80 at *c*.
5·04	9·25	4·21		13·11	11·60 at *d*.
3·84	12·91	9·07		22·18	15·20 at *e*.
4·12	7·65	3·53		25·71	bottom of canal, distance 2·16
10·49	3·92		6·57	19·14	21·00 at *f*.
12·96	3·03		9·93	9·21	27·00 at *g*.
44·05	53·26				
	44·05				
diff.	9·21	the same as the last of the reduced levels.			

In this level book it will be seen that the differences 2·15 and 6·75, in the column marked Fall, are added together, making 8·90, thus giving the fall at *c*, in the column marked Reduced Levels: to this sum the succeeding falls are added, one by one, till we get the fall 25·71 at the bottom of the canal, which is the lowest point. Then the differences in the column marked Rise, are subtracted successively from 25·71 for the falls at *f* and *g*; the latter of which is 9·21, the total fall from *a* to *g*, which, agreeing with the difference of the sums of the back and fore sights, shews the truth of the castings. The last column shews the distances of the several points *b*, *c*, &c., from *a*, in chains, with other remarks.

DATUM LINE.

The section might be plotted by laying off the distances in the last column in the preceding level book on a horizontal line, and setting off their corresponding numbers of feet, in the column marked Reduced Levels, perpendicularly below the line: but it is found inconvenient in practice to plot a section in all cases after this method, as in extensive operations the reduced levels would repeatedly fall above anu below the line in question, and thus confuse the operation; therefore a line AG, called "the datum line," is assumed at 100, 200 feet, &c., below the first station *a*; thus making that line always below the sectional line *a f*, of which a clearer view may be obtained.

In the following practical level book the rise or fall is re-

spectively added to, or subtracted from, the assumed distance of the datum line, and the next rise or fall again added to, or subtracted from, the sum or difference:—thus 2·15, being a fall, is subtracted from 100 (the assumed distance of the datum line) leaving 97·85 feet, the height of the ground at *b*; the next fall 6·75 is then subtracted from 97·85, leaving 91·10 feet for the height at *c*; and so on to 3·53, which is the last fall:—the next 6·57, being a rise, is added, as well as 9·93;—thus the last reduced level is 90·79 feet, which taken from the datum 100 leave 9·21 feet, agreeing with the differences of the sums of the back and fore sights, and of the sums of the rises and falls, and shewing the work of casting to be correct. Thus are obtained a series of vertical heights to be set off perpendicularly to the datum line, through the upper extremities of which the sectional line must be drawn.

PRACTICAL LEVEL BOOK.

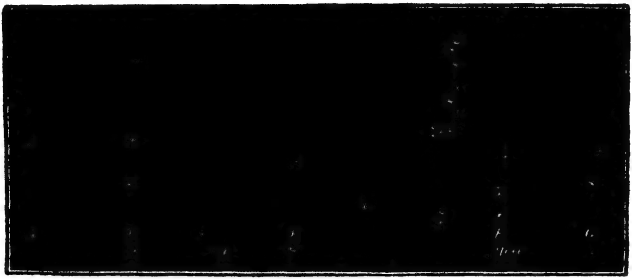
(Datum line 100 feet below the bench mark at A.)

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				100·00 p		
3·50	5·65		2·15	97·85	4·60	B M on road to lime kilns.
4·10	10·85		6·75	91·10	7·80	
5·04	9·25		4·21	86·89	11·60	
3·84	12·91		9·07	77·82	15·20	Bottom of canal, distant 2·80 chains.
4·12	7·65		3·53	74·29	......	
10·49	3·92	6·57		80·86	21·00	
12·96	3·03	9·93		90·79	27·00	to B M at *g*.
44·05	53·26	16·50	25·71	100·00		
	44·05		16·50			
	9·21	diff. =	9·21	= 9·21		diff. between last reduced level and datum.

In laying down the sectional line from the above columns of reduced levels and distances, the former are always taken from a much larger scale than the latter, otherwise the undulations on the surface of the ground would in many cases, be hardly perceptible.

Draw the horizontal line A G, setting off the distances A B, A C, &c., as in the column of distances, that is A B = 4·60

chains, A C = 7·80, &c., then draw A a = 100 feet, perpendicular to A G and parallel to A a draw B b, C c, &c., setting



off their heights 97·85, 91·10, &c., respectively, from the column of reduced levels, and through the points a, b, c, &c., draw the required sectional line a g.

NOTE. The above operations, though extremely simple, require great care, otherwise, in extensive works of this kind, errors creep in imperceptibly, to check which the agreement of the differences in the level book is essential.

LEVELS FOR THE FORMATION OF A SECTION.

In this case it is required to take the levels of a line of country, where the ground plan is already made, and the line of section determined upon, and marked out on the plan. Here, in addition to what is required in running or check levels, the distances to the several stations of the levelling staves from the starting point must be measured.

Two additional assistants are required in this case to measure the distances of the stave stations along the lines while the operation of levelling goes on, which is the same in every respect as that already described, excepting that, in this case, the operation is conducted upon a line, on the surface plan, a copy of which must be in the surveyor's possession to direct him, and the distances of the several stave stations must be noted in the level book, in the column marked "Distances."

The following is the level book of an example, shewing the manner of keeping it, and also the method of reducing the levels, to obtain the actual heights of each station above the datum line, which is placed 100 feet below the starting point, for convenience of drawing the section. The whole operation being similar to that already given at page 123, excepting that here we give the particular manner of performing the several parts of the field work, in order that it may be clearly understood by those who are unacquainted with the subject, as it is presumed that, in a short time, railways will become the common means

of transit, both for passengers and goods, throughout every country of the civilized world.

THE LEVEL BOOK FOR PLOTTING THE SECTION.

(Datum 100 feet below the station A.)

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances	Remarks.
feet.	feet.	feet.	feet.	feet.	links.	
				100·00 B		
13·71	7·88	5·83		105·83	519	B.M. side of road.
9·40	16·30		6·90	98·93	1315	
3·87	11·71		7·84	91·09	1542	
2·63	12·41		9 78	81·31	1850	
14·62	0·95	13·67		94·98	2358	
17·00	1·45	15·55		110·53	2698	
10·66	15 40		4·74	105 79	3357	
2·87	17·00		14·13	91 66	3758	
3·40	10·32		6·92	84 74	3976	
5·73	2·24	3·49		88·23	5077	
16 54	0·85	15 69		103 92	5904	
16·08	0·89	15·19		119·11	6124	
14·56	0·73	13·83		132·94	6437	
10·36	14·06		3 70	129 26	7407	
9·84	1·36	8·48		137 72	8309	
9·80	7·00	2·80		110 52	9303	
2·30	10·96		8·66	131 86	——	Centre of road at 215 links.
10 96	14·46		3·50	128·36	9679	
2·08	15·05		12 97	115 39	9936	
1 75	16·58		14·83	100 56	10164	
1·84	17·10		15·26	85 30	10576	
0 00	7·43		7·43	77·87	11423	Forward ⊙ at corner of wood.
5·38	3·50	1·88		79·75	13066	
8·50	4·50	4·00		83·75	14954	
5·30	1·36	3·94		87·69	15650	
10·20	9·40	0·80		88·49	17345	
6·86	0·40	6·46		94 95	19135	
11·00	3·96	7 04		101·99	19359	
11·80	3·53	8 27		110·26	19631	
10·53	2·68	7·85		118·11	19841	Forward ⊙ at end of wood.
8·82	1·38	6·84		124·95	20561	
8·76	2·20	6·56		131·51	21671	
14 00	14·50		0 50	131·01	——	Road at 450 links.
14 50	4·32	10·18		141·19	22710	
9·14	1·00	8·14		149·33	23221	
304·19	254·86	166·49	117·16	100·00		
254·86		117·16				
49·33	—	49 33	—	49·33		Difference between Datum and last Reduced level, or height of B above A.

The several differences of the sums of the back and fore sights, of the sums of the rises and falls, and of the last reduced

level and the datum, exactly agreeing, proves the accuracy of the arithmetical operation in the preceding level book, all these differences being 49·33 feet, which is the height of the last station above the first.

It is advisable for the surveyor to reduce the levels in the field as he proceeds, as it will occupy very little time and can be easily done while the staffman is taking a new position. The surveyor will thus be enabled to detect with the eye if he is committing any glaring error; for instance, inserting a number in the column of rises, when it ought to be in that of falls, the surface of the ground at once reminding him that he is going downward instead of ascending.

It is seldom the case in practice that the instrument can be placed precisely equi-distant from the back and fore staves, on account of the inequalities of the ground, ponds, &c.; it would appear, therefore, to be necessary, to make our results perfectly correct, to apply to each observation the correction for curvature and refraction as explained at page 118: this, we believe, is seldom done unless in particular cases, where the utmost possible accuracy is required, on account of the smallness of such correction, as may be seen by referring to the table at the end of the book, where this correction for 11 chains is shewn to be no more than $\frac{1}{100}$ part of a foot; and as the difference in the distances between the instrument and the fore and back staves can in no case equal that sum, it is evident that such correction may be safely disregarded in practice. Besides it is not necessary to have the level placed directly between the staves while making observations, as it is frequently inconvenient to do so, for reasons just given, nor does a deviation from a line of the staves, in this respect, in the least affect the accuracy of the result.

The distances in the sixth column of the level book are assumed to be horizontal distances, and in measuring them, care should be taken that they are as nearly such as possible, or they must be afterwards reduced thereto, otherwise the section will be longer than it ought to be. For the purpose of assisting the surveyor in making the necessary reduction from the hypothenusal to the horizontal measure, when laying down the section, a table is given in *Baker's Land and Engineering Surveying*, page 146, shewing the reduction to be made on each chain's length for the following quantities of rise, as shewn by the reading of the staves.

NOTE. For extensive information on this subject see *Baker's Land and Engineering Surveying*, where an engraved plan and section, adapted to this example, are given at the end of the work.

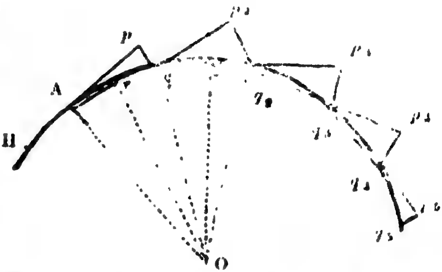
THE METHOD OF LAYING OUT RAILWAY CURVES ON THE GROUND.

In railway practice, the curve adopted is always *an arc of a circle*, to which the straight portions of the railway are tangents at each extremity of the arc. Sometimes the curve consists of two, three, or more circular arcs with their concavities turned in the same or different directions, as in *the compound and serpentine curves.*

PROBLEM I.

To lay out a railway curve on the ground by the common method.

CASE I.—Let H A, $q_4\,q_5$ be the tangental portions of a railway, the extremities A and q_4 of which are required to be united by the circular curve A q_4, to which H A, $q_5\,q_4$ shall be tangents; the radius of the curve being supposed to be previously determined.



Let the radius in this case be 80 chains or one mile; prolong the tangent B A a distance A p = 1 chain: then opposite 80 in table No. 2, at the end of the book, is found 4·95 inches = $p\,q$, which set off at right angles to A p, thus giving the first point in the curve. In the direction A q, measure $q\,p_2$ = 1 chain, and set off $p_2\,q_2$ = twice $p\,q$ = 4·95 × 2 = 9·9 inches, at right angles to $q\,p_2$; then q_2 is the second point in the curve. This last operation must be repeated till the curve shall have been set out to the point q_4. Lastly $q_4\,p_5$ being measured = 1 chain, in the direction $q_3\,q_4$, the offset $p_5\,q_5$ will be found = 4·95 inches = the first offset $p\,q$, thus proving the accuracy of the work. In this manner the operation is conducted, whatever be the length of the curve.

CASE II.—Let A O = r, and s = A p = $q\,p_2$ = &c., which may be either less or greater than one chain; then the general length of the first and last offsets $p\,q$, $p_5\,q_5$ is $\frac{s^2}{2r}$, and the length of each of the other offsets is $\frac{s^2}{r}$, or twice the first or

last offset; but the length of the offsets given in the table is represented by $\frac{1}{2r}$; therefore, if $A p$, $q p_2$, &c., be taken as 2, 3, 4, &c., chains, the value of $\frac{1}{2r}$ must be multiplied by $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, &c., respectively to find $p\ q$, and the result, in each case, multiplied by 2 for each of the offsets $p_2\ q_2$, $p_3\ q_3$, &c. In this manner the curve may be set out more speedily, and with less liability to error, on account of the less number and greater length of the lines required in the operation.

EXAMPLE.

Let $A\ O = r = 120$ chains, and $\delta = 4$ chains; then $\frac{\delta^2}{2r} = \frac{1}{2r} \times 16 = 3{\cdot}3 \times 16 = 52{\cdot}8$ inches 4 feet 4·8 inches $= p\ q$; whence 4 feet 8 inches $\times 2 = 4{\cdot}8$ feet 9·6 inches $= p_2\ q_2$, $p_3\ q_3 =$ &c.

NOTE 1. When the curve has been correctly set out, as in Case II., the intermediate stumps may be put in at the end of every chain, if required, by the method given in Case I. The distances of the intermediate stumps, thus put in, will not, in most cases, exceed a fraction of an inch; because the lengths of the offsets $p\ q$, $p_2\ q_2$, &c., is so small, that the curvilinear lengths $A\ q$, $q\ q_2$, &c., can never greatly exceed those $A\ p$, $q\ p^2$, &c.

NOTE 2. The method given in Case II., is sufficiently accurate when δ does not exceed $\frac{1}{10}$ of the radius of the curve. Besides, at the closing point of the curve, as at q_4, the distance $q_3\ p_4$ is most commonly less or greater than δ. Let $q_3\ p_4 = d$; then the offset $p_4\ q_4$, at the end of the curve is $= \frac{(\delta + d)d}{2r}$; and, when $\delta = 1$ chain, $p^4\ q^4 = \frac{(1 + d)d}{2r}$; or the tabular number for the given radius must be multiplied by $(\delta + d)d$, or by $(1 + d)d$, according as $A\ p$, $q\ p_2$, &c., is taken $= \delta$ chains or 1 chain, to give the last offset $p_4\ q_4$; $\frac{1}{2}$ of which is $= p_5\ q_5$, the offset to the tangent $q_4\ q_5$.

EXAMPLE.

Let $r = 120$, and $\delta = 4$ chains, as in the last example, and let $q_3\ p_4 = d = 2{\cdot}68$ chains; then $p_4\ q_4 = \frac{1}{2r} \times (\delta + d)\ d = 3{\cdot}3 \times (4 + 2{\cdot}68) \times 2{\cdot}68 = 59{\cdot}07$ inches; $\frac{1}{2}$ of which, viz., $29{\cdot}535$ inches is $= p_5\ q_5$.

NOTE 3. When δ exceeds $\frac{1}{10}$ of the radius r of the curve, the following formula ought to be used for finding the offsets.

$$p\ q = r - \sqrt{r^2 - \delta^2},$$

$$\text{and } p^2\ q^2 = \text{\&c.} = \frac{\delta^2}{\sqrt{r^2 - \frac{1}{4}\delta^2}}.$$

See *Baker's Land and Engineering Surveying*, page 161.

NOTE 4. By this method the greater part of both British and foreign rail-

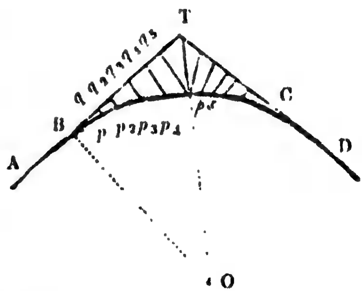
way curves have been laid out. It was invented by the author about 27 years ago, when the Stockton and Darlington Railway was laid out, and eagerly adopted by engineers, as it involves very little calculation, and does not require the use of a theodolite. It is, however, defective in practice, on account of its requiring so very many short lines connected together, as errors will unavoidably creep in and multiply, and more especially so where the ground is rough; thus the curve has frequently to be retraced several times before it can be got right; hence the author prepared the methods in the following Problem.

Problem II.

To lay out a railway curve on the ground, by offsets from its tangents, no obstructions being supposed to prevent the use of the chain on the convex side of the curve.

Case I.—*When the length of the curve does not exceed $\frac{1}{4}$ of its radius.*

Let A B, D C be straight portions of a railway, the points C and B being required to be joined by a circular curve B C, to which A B, D C shall be tangents, the radius B O of the curve being supposed to be previously determined from an accurate plan of the intended railway.



Range the tangents A B, D C till they meet at T; and let the radius B O = 80 chains = 1 mile; measure on B T the distance B q = 1 chain; and, at right angles to B T, lay off the offset $q\,p$ = 4·95 inches, by Table No. 2, as in Problem I.; then p is the first point in the curve. Next measure $q\,q_2$ = 1 chain, and lay off the offset $p_2\,q_2 = 4{\cdot}95 \times 4$ for the second point in the curve. The successive offsets, at the end of every chain, being 4, 9, 16, &c., or 2^2, 3^2, 4^2, &c., times the first offset $p\,q$, which may also be found opposite the given radius in the Table No. 2., as in Prob. I.

When the offsets have been thus laid out, till the last one $q_5\,p_5$ falls little short of T; lay off the same offsets on T C as were laid off in B T, but in an inverted order, making the first distance on T C = T q_5; thus completing the curve to C.

Note. It can rarely happen in practice that the last offsets, from both tangents, will meet at the middle point p_5 of the curve, as shewn in the figure; but will either intersect one another or fall short of the middle point; but this is a matter of no consequence.

EXAMPLE.

Let the radius of the curve be 160 chains, required the offsets at the end of every chain, from the tangent to the curve.

$$
\begin{array}{lcl}
p\ q \text{ (per Table No. 1.)} & = & 2{\cdot}475 \text{ inches.} \\
p_2\, q_2 = 2{\cdot}475 \times 4 & = & 9{\cdot}9 \text{ ——} \\
p_3\, q_3 = 2{\cdot}475 \times 9 & = & 22{\cdot}275 \text{ ——} \\
p_4\, q_4 = 2{\cdot}475 \times 16 & = & 39{\cdot}6 \text{ ——} \\
\text{\&c.} = \text{\&c.} & = & \text{\&c.}
\end{array}
$$

CASE II.—*To lay out the curve when it is any required length.*

In a long curve (of which there are some more than two miles in length) the tangents, if prolonged to their point of meeting, would necessarily fall at a great distance from the curve, thus giving an inconvenient length to the offsets, which in practice should never exceed two chains. To remedy this inconvenience the curve must be divided into two or more parts, by introducing one or more additional tangents, thus the offsets may be confined within their proper limits. Thus the tangent T C may, in this case, be extended, another tangent applied, and the offsets laid off, thus repeating the operation of Case I. a second time: if the curve be not yet completed, the operation may be repeated a third, fourth, &c., time, till it be completed.

NOTE. For a complete developement of this important subject, see *Baker's Land and Engineering Surveying*, Part II., Chap. II., where two other methods of laying out railway curves are given; also methods of laying out compound, serpentine, and deviation curves, with original formulæ; all of which methods, as well as the two already given, were first drawn up by the author. See, also, page 174 of the work above referred to, where a short history of the invention is given.

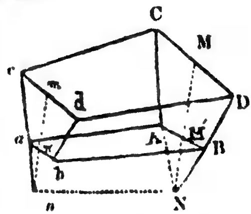
CONTENTS OF RAILWAY CUTTINGS, &c.

TABLES.

The *General Earthwork Table*, in conjunction with *Two Auxiliary Tables*, on the same sheet, in *Baker's Railway Engineering*, or the number for the slopes in *Bidder's Table*, are applicable to all varieties of ratio of slopes and widths of formation level in common use; and with the help of Barlow's table of square roots, these tables will apply to sectional areas, with all the mathematical accuracy that can be attained, with very little more calculation than adding the contents between every two cross-sections, as given by the *General Table*.—The contents in the General Table are calculated to the nearest unit,

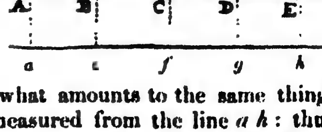
as are also those in the Auxiliary Table, No. 2, which is for the decimals of feet in the depths. The Auxiliary Table, No. 1, shews the depths of the meeting of the side-slopes below the formation-level, with the number of cubic yards to be subtracted from the contents of the General Table for each chain in length, for eight of the most common varieties of ratio of slope.

The following diagrams and explanations will further illustrate the method of taking the dimensions of railway cuttings, preparatory to using the above named tables.



Let A B D C *c a b d*, be a railway cutting, of which A B D C, *a b d c* are the cross sections, A B = *a b* = width of formation level, M M′, *m m′* the middle depths of the two cross-sections; the side-slopes A C, D B, *a c*, *b d*, when prolonged two and two, will intersect at N and *n*, at which points the prolongations of M M′, *m m′* will also meet, thus constituting a prism A B N *n a b*, the content of which is to be deducted from the whole content, given by the General Table, by means of the Table No. 1.; in which the depth M′ N = *m′ n* is also given, as already stated, to several varieties of slope and bottom width.

To place this subject in a more practical point of view, let the annexed figure represents a longitudinal and vertical section of a cutting, passing through the middle A E of the formation level. H I, the line of the rails, and *a h*, the line on which the slopes, if prolonged, would meet. It will be seen that the cutting A *b c d* E commences and runs out on the formation level A E, and that the depth A *a* = B *e* = C *f* = &c., is to be added to the several depths B *b*, C *c*, D *d* of the cutting, the first and last depth at A and E being each = 0; or, what amounts to the same thing, the several depths must be measured from the line *a h* : thus A *a*, *b e*, *c g*, &c., are the depths to be used. And since the depth A *a* is given in Table No. 1, for all the most common cases, or it may be readily found by calculation for all cases, (see *Railway Engineering*), the line corresponding to *a h* must, therefore, be ruled on the railway section, at the proper distance



below A E, from which the several depths must be measured; or the vertical scale may be marked with Indian ink, (which may be readily rubbed off) at the same distance, and this mark may then be applied to the formation level A E, for the purpose of measuring the several depths.—In the case of an embankment, the line for the several depths must be placed at a like distance above the formation level.

Problem I.

The severals depths of a railway cutting to the meeting of the side slopes, its width of formation level, and the ratio of the slopes being given, to find the content of the cutting in cubic yards, from the Tables referred to, the distances of the depths being one chain each.

Rule.—Take the several quantities, corresponding to every two succeeding depths of a cutting, or embankment, measured to the meeting of the side slopes, at the distance of 1 chain each, from the General Table in *Baker's Railway Engineering*, and multiply their sum by the ratio of the slopes; from the product subtract the cubic yards, corresponding to the given bottom-width and ratio of slopes from Table No. 1., multiplied by the whole length of the cutting, and the remainder will be the content of the cutting in cubic yards.

But, when the distances of the depths are greater or less than 1 chain, the quantities of the General Table must be multiplied by their respective distances.—And, when the distances are given in feet, the quantities must be multiplied by those distances, and the final result divided by 66 for the content in cubic yards, as in the following

EXAMPLES.

1. Let the depth of the railway cutting or embankment to the meeting of the side-slopes, at the end of every chain, be as in the following table, the bottom-width 30 feet, and the ratio of the slopes as 2 to 1; required the content in cubic yards.

Dist. in chains.	Depth in feet.	Qnts per G. Table.
0	10	
1·00	33	1238
2·00	39	3175
3·00	35	3350
4·00	10	1355
For slope 1 to 1 ..		9128
		2
For slope 2 to 1 .		18256
Subtract 275 × 4	=	1100
Content in cubic yds.	=	17156

Note. In the annexed table the quantity 1238, corresponds to the depths 10 and 33 feet, in the General Table; the quantity 3175 to the depths 33 and 39, and so on for the succeeding depths. By the Auxiliary Table No. 1. it will be seen, that the depth to be added below the formation level, for the given width and ratio of slopes, is 7·50 = 7½ feet, therefore, the cutting begins and ends with a depth of 10 — 7½ = 2½ feet. The corresponding number of cubic yards to be deducted for each chain

in length, is multiplied by 4 chains, the whole length of the cutting, thus giving the whole quantity to be deducted, the remainder being the true content in cubic yards of the cutting.

2. The several depths of a railway cutting to the meeting of the side slopes are as in the annexed table, the bottom width being 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting.

Dist. in chains.	Depths in feet.	Products for Dist. greater than 1 chain.	Total quantities.
0	10		
1·00	16	. . . . .	420
2·00	20	. . . . .	705
4·00	25	1234 × 2	2486
5·00	32	. . . . .	1996
7·00	39	3091 × 2	6182
8·00	45	. . . . .	4319
10·00	50	5520 × 2	11040
12·00	40	4971 × 2	9942
13·00	30	. . . . .	3015
14·46	10	1059 × 1·46	1546
For side slopes 1 to 1			41741
For side slopes $\frac{1}{2}$ to 1			20870
For side slopes $1\frac{1}{2}$ to 1			62611
		366·67 × 14·46 =	5302
Content in cubic yards			57309

NOTE. When any of the distances between two succeeding depths is greater or less than 1 chain, the corresponding quantity from the General Table must be multiplied by that particular distance; as the distances between the depths 20 and 25, and between 32 and 39, &c., the distance being 2 chains. The last distance, viz., that between 30 and 10, is 1·46, in this case 2 figures must be cut off for decimals, after multiplying.

3. Let the depths of a railway cutting to the meeting of the side slopes, and their distances in feet be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content in cubic yards.

Dist. in feet.	Depths in feet.	Quantities multiplied by length.	Total quantities.
0	37		
90	50	4660 × 90	419400
178	61	7554 × 88	664752
278	39	6210 × 100	621000
For slopes 1 to 1			1705152
For slopes $\frac{1}{2}$ to 1			852576
For slopes $1\frac{1}{2}$ to 1			2557728
		366·67 × 278 =	101933
			66)2455795
Content in cubic yards			37209

NOTE. When the distances are in feet the quantities from General Table must be respectively multiplied by their distances, the quantity from Table No. 1, by the whole distance, and the result divided by 66, the feet in 1 chain, for the content in cubic yards.

PROBLEM II.

CASE I.—*The areas of two cross sections of a railway cutting to the intersection of the side slopes, its length in chains, bottom width, and ratio of the slopes are given; required the content of the cutting in cubic yards.*

RULE.—With the square roots of the given areas as depths,

find the content from the General Table, as in the last Problem, from which subtract the quantity answering to the given width, and the ratio of side-slopes from Table No. 1, and the remainder, being multiplied by the length, will be the content required.

NOTE. If the length be given in feet, proceed as in Example 3, last Problem.

EXAMPLE.

1. Let the two sectional areas of a cutting be 4761 and 1296 square feet, the bottom width 36 feet, the length 3·25 chains, and the ratio of the side slopes 2 to 1; required the content in cubic yards.

$\left.\begin{array}{l}\sqrt{4761} = 69 \\ \sqrt{1296} = 36\end{array}\right\}$ content per General Table ... 6959

For bottom width 36 and slopes 2 to 1, per Table No. 1 396

Content for 1 chain in length.................... 6563

$3\frac{1}{4}$

19689

1641

Content for 3·25 chains.........................21330 cubic yds.

CASE II.—*In measuring contract work, where great accuracy is required, the $\frac{1}{100}$ths of a foot, or second decimals, must be used in the calculation, by taking for them $\frac{1}{10}$th of their respective quantities in Table No. 2.*

EXAMPLE.

The areas of seven cross sections of a railway cutting to the meeting of the side slopes, and their distances are as in the annexed table; the bottom width is 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the cubic yards in the cutting.

Dist. in chains.	Areas in sq. feet.
0	2727
2·00	3136
6·00	4221
9·00	4100
14·00	5141
16·00	3759
18·00	2161

Ans. The content, per General Table, and Table No. 2, is 172318 cubic yards, from which the quantity corresponding to the given bottom width and ratio of slopes × by the whole length, viz. 275 × 18 = 4950 cubic yards must be deducted, which leaves 167568 cubic yards, the content required.

NOTE. For further explanations and numerous examples of the methods of finding the contents of earthwork, see *Baker's Land and Engineering Surveying.*

TABLE No. 1.—THE AREAS OF SEGMENTS OF CIRCLES, DIAMETER UNITY.

Height.	Area Segment.	Height.	Area Segment.	Height	Area Segment.
·001	·000042	·050	·014681	·099	·040276
·002	·000119	·051	·015119	·100	·040875
·003	·000219	·052	·015561	·101	·041476
·004	·000337	·053	·016007	·102	·042080
·005	·000470	·054	·016457	·103	·042687
·006	·000618	·055	·016911	·104	·043296
·007	·000779	·056	·017369	·105	·043908
·008	·000951	·057	·017831	·106	·044522
·009	·001135	·058	·018296	·107	·045139
·010	·001329	·059	·018766	·108	·045759
·011	·001533	·060	·019239	·109	·046381
·012	·001746	·061	·019716	·110	·047005
·013	·001968	·062	·020196	·111	·047632
·014	·002199	·063	·020680	·112	·048262
·015	·002438	·064	·021168	·113	·048894
·016	·002685	·065	·021659	·114	·049528
·017	·002940	·066	·022154	·115	·050165
·018	·003202	·067	·022652	·116	·050804
·019	·003471	·068	·023154	·117	·051446
·020	·003748	·069	·023659	·118	·052090
·021	·004031	·070	·024168	·119	·052736
·022	·004322	·071	·024680	·120	·053385
·023	·004618	·072	·025195	·121	·054036
·024	·004921	·073	·025714	·122	·054689
·025	·005230	·074	·026236	·123	·055345
·026	·005546	·075	·026761	·124	·056003
·027	·005867	·076	·027289	·125	·056663
·028	·006194	·077	·027821	·126	·057326
·029	·006527	·078	·028356	·127	·057991
·030	·006865	·079	·028894	·128	·058658
·031	·007209	·080	·029435	·129	·059327
·032	·007558	·081	·029972	·130	·059999
·033	·007913	·082	·030526	·131	·060672
·034	·008273	·083	·031076	·132	·061348
·035	·008698	·084	·031629	·133	·062026
·036	·009008	·085	·032186	·134	·062707
·037	·009383	·086	·032745	·135	·063389
·038	·009763	·087	·033307	·136	·064074
·032	·010148	·088	·033872	·137	·064760
·040	·010537	·089	·034441	·138	·065449
·041	·010931	·090	·035011	·139	·066140
·042	·011330	·091	·035585	·140	·066833
·043	·011734	·092	·036162	·141	·067528
·044	·012142	·093	·036741	·142	·068225
·045	·012554	·094	·037323	·143	·068924
·046	·012971	·095	·037909	·144	·069625
·047	·013392	·096	·038496	·145	·070328
·048	·013818	·097	·039087	·146	·071033
·049	·014247	·098	·039680	·147	·071741

When the tabular height exceeds the heights given in this Table the segment must be divided into two equal parts and their common versed sign found by Prob. VI, page 26. The tabular heights will then fall within this Table, whence the area of the whole segment may be found.

TABLES OF OFFSETS FOR RAILWAY CURVES, AND CORRECTION OF LEVELS FOR CURVATURE, ETC.

No. 2.—Offsets at the end of the first chain from tangent point of railway curves.

Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.
40	9·9000	64	6·1875	88	4·5000	120	3·3000
41	9·6588	65	6·0923	89	4·4496	122	3·2459
42	9·4285	66	6·0000	90	4·4000	124	3·1935
43	9·2093	67	5·9104	91	4·3516	125	3·1680
44	9·0000	68	5 8235	92	4·3043	126	3·1428
45	8·8000	69	5·7391	93	4·2581	128	3·0937
46	8·6087	70	5·6571	94	4·2128	130	3·0461
47	8·4255	71	5·5774	95	4·1684	132	3·0000
48	8·2500	72	5·5000	96	4·1250	134	2·9552
49	8·0816	73	5·4246	97	4·0825	135	2·9333
50	7·9200	74	5·3513	98	4·0408	136	2·9117
51	7·7647	75	5·2800	99	4·0000	138	2·8645
52	7·6154	76	5·2105	100	3·9600	140	2·8285
53	7·4717	77	5·1428	102	3·8824	142	2 7887
54	7 3333	78	5·0769	104	3·8077	144	2·7500
55	7·2000	79	5·0126	105	3·7714	145	2·7310
56	7·0714	80	4 9500	106	3·7358	146	2·7123
57	6·9473	81	4·8889	108	3·6667	148	2·6756
58	6·8276	82	4·8292	110	3 6000	150	2·6400
59	6·7118	83	4·7711	112	3·5352	152	2·6052
60	6·6000	84	4·7143	114	3·4736	154	2·5714
61	6·4918	85	4·6588	115	3·4435	155	2·5548
62	6·3871	86	4·6046	116	3·4138	156	2·5384
63	6·2857	87	4·5517	118	3·3559	158	2·5063

TABLES OF CORRECTIONS FOR CURVATURE, ETC.

No. 3. Difference between apparent and true level for distances in chains. Correction in decimals of feet.

Distance in chains.	For curvature and refraction.	Distance in chains	For curvature and refraction.
3½	·001	14	·017
4	·002	14½	·019
4½	·002	15	·020
5	·003	15½	·021
5½	·003	16	·023
6	·004	16½	·024
6½	·003	17	·026
7	·004	17½	·027
7½	·005	18	·029
8	·006	18½	·031
8½	·006	19	·033
9	·007	19½	·034
9½	·008	20	·036
10	·009	20½	·038
10½	·009	21	·039
11	·011	21½	·041
11½	·012	22	·043
12	·013	22½	·046
12½	·014	23	·047
13	·016	23½	·049
13½	·016	24	·051

No 4.— Difference between apparent and true level for distances in miles. Correction in feet and decimals.

Distance in miles	For curvature and refraction.	Distance in miles	For curvature and refraction.
¼	·03	10	57 17
½	·15	10½	61·30
¾	·32	11	69·16
1	·58	11½	75·59
1½	1·29	12	82·29
2	2·29	12½	89·29
2½	3·57	13	96·58
3	5·14	13½	104·14
3½	7·00	14	112·00
4	9·15	14½	120·15
4½	11 62	15	128·57
5	14·29	15½	137·29
5½	17·30	16	146·29
6	20·58	16½	155·57
6½	24·15	17	165·15
7	28·01	17½	175·00
7½	32 16	18	165·14
8	36·59	18½	195·59
8½	41·31	19	206·29
9	46·30	19½	217·29
9½	51·60	20	228·60

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